Algebra 1 Unit 10: Exponential Functions Notes

Day 3 – Transformations of Exponential Functions

Transformations of exponential functions is very similar to transformations with quadratic functions. Do you remember what a, h, and k do to the quadratic function?

A: _____ H: _____ K: _____

Summary of Exponential Transformations

The general form of an exponential function is:

$$f(x) = a(b)^{x-h} + k.$$

***When your graph is shifted vertically, the y-intercept becomes a + k.**
***When the graph is shifted vertically, the asymptote becomes y = k.**

If **a** is **negative**, the graph... reflect

If h is **positive**, the graph... right
In the equation, I would see... − h (hypo.)
If h is **negative**, the graph... Left
In the equation, I would see... +h

$$y = a(b)^{x-h} + k$$

If **a** is **between 0 and 1**, the graph... Shrink
Grows _____

If **a** is **greater than 1**, the graph... Stretch
Grows _____

If **b** is **greater than 1**... grow
If **b** is **between 0 & 1**... decay

If k is **positive**, the graph...
If k is **negative**, the graph...
Asymptote: y = k

Algebra 1 Unit 10: Exponential Functions Notes

Example: Find the y-intercept and asymptote of the following equations:

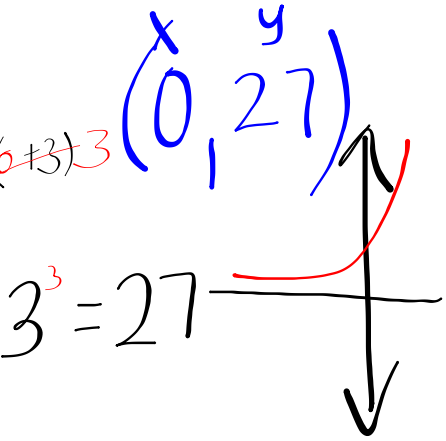
A. $f(x) = 3^x \rightarrow f(x) = 3^{x+3}$

y-intercept: asymptote:

$y = 3^{x+3}$ Calc: $y = 3 \wedge (x+3)$

x	y
0	27

$y = 3^{(0+3)} = 3^3 = 27$ (0, 27)



B. $y = \frac{1}{2}(5)^x \rightarrow y = \frac{1}{2}(5)^x - 4$

y-intercept: asymptote:

C. $y = 3(0.4)^x \rightarrow y = 3(0.4)^x + 8$

y-intercept: asymptote:

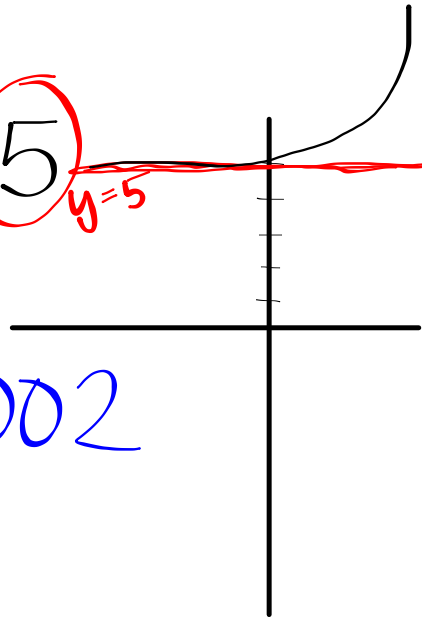
D. $f(x) = 4^x \rightarrow f(x) = 4^{x-6} + 5$

y-intercept: asymptote:

Calc: $y = 4 \wedge (x-6) \rightarrow +5$ $y = 5$

Table

x	y
0	5.0002



y-int:

1. Calc on table

x	y
0	_

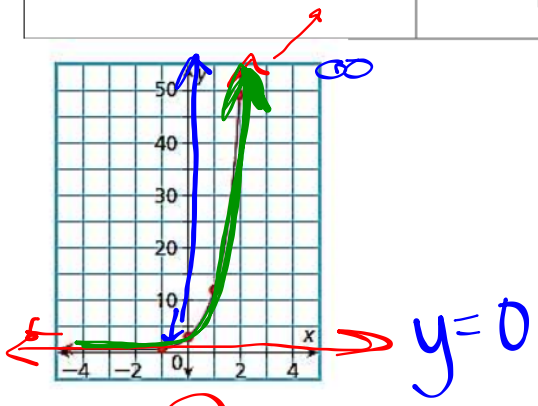
2. Substitute $x=0$
3. Look @ graph where it crosses y-axis

Day 4 – Characteristics of Exponential Functions

As you can hopefully recall, you learned about characteristics of functions in Unit 2 with linear functions and Unit 5 with quadratic functions. We are going to apply the same characteristics, but this time to exponential functions.

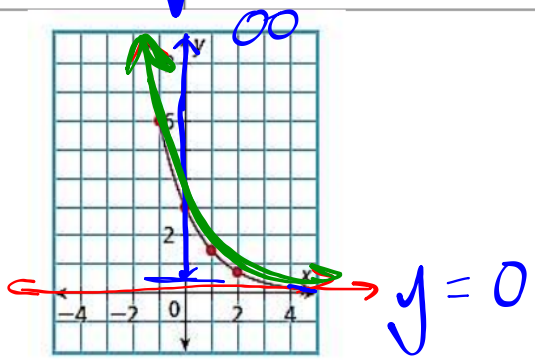
Domain and Range

Domain		
Define: All possible values of x	**Think:** How far left to right does the graph go?	**Write:** Smallest x ≤ x ≤ Biggest x *use < if the circles are open*
Range		
Define: All possible values of y	**Think:** How far down to how far up does the graph go?	**Write:** y < highest y value (opens down) y > lowest y value (opens up)



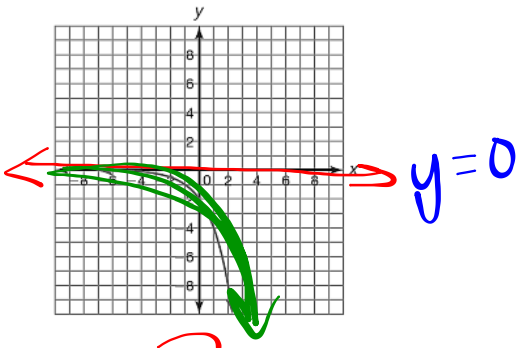
Domain: $\mathbb{R}$

Range: $y > 0$



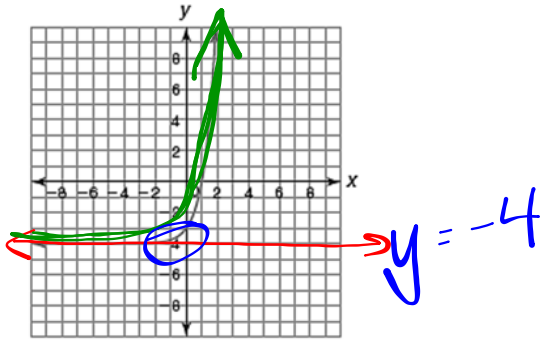
Domain: $\mathbb{R}$

Range: $y > 0$



Domain: $\mathbb{R}$

Range: $y < 0$



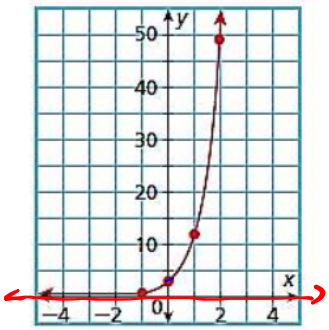
Domain: $\mathbb{R}$

Range: $y > -4$

Algebra 1 Unit 10: Exponential Functions Notes

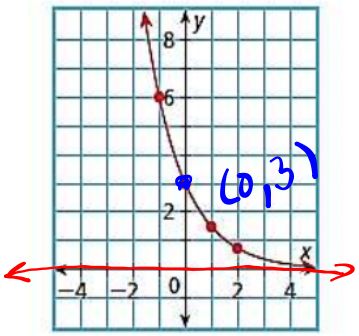
Intercepts and Zeros

Y-Intercept		
Define: Point where the graph crosses the y-axis	**Think:** At what coordinate point does the graph cross the y-axis?	**Write:** (0, b) (0, #)
X-Intercept		
Define: Point where the graph crosses the x-axis	**Think:** At what coordinate point does the graph cross the x-axis?	**Write:** (a, 0)
Zero		
Define: Where the function (y-value) equals 0	**Think:** At what x-value does the graph cross the x-axis?	**Write:** x = ____



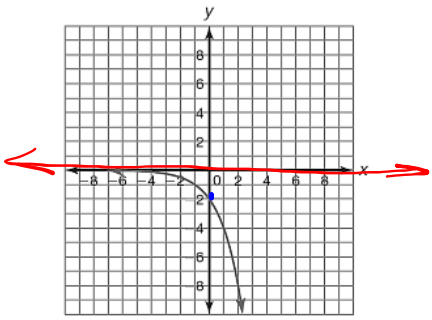
X-intercept: none Zero: none

Y-intercept: (0,4)



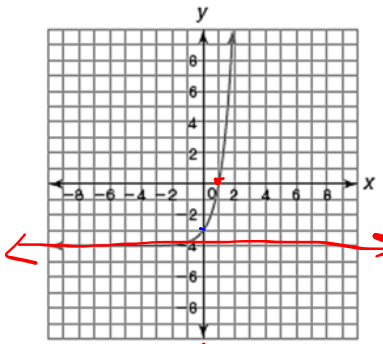
X-intercept: none Zero: none

Y-intercept: (0,3)



X-intercept: none Zero: none

Y-intercept: (0,-2)



X-intercept: (1,0) Zero: x = 1

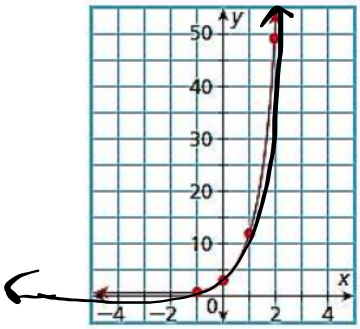
Y-intercept: (0,-3)

Algebra 1 Unit 10: Exponential Functions Notes

Extremas and Asymptotes

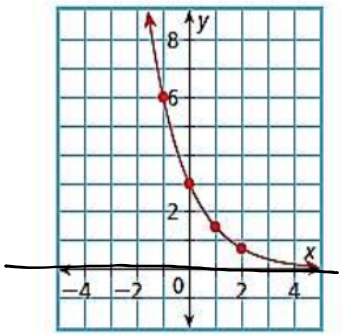
Maximum		
Define: Highest point of a function.	**Think:** What is my highest point on my graph?	**Write:** y =
Minimum		
Define: Lowest point of a function.	**Think:** What is the lowest point on my graph?	**Write:** y =
Asymptotes		
Define: A line that the graph get closer and closer to, but never touches or crosses.	**Think:** What values does my graph begin to flat line towards?	**Write:** y =

change in direct.



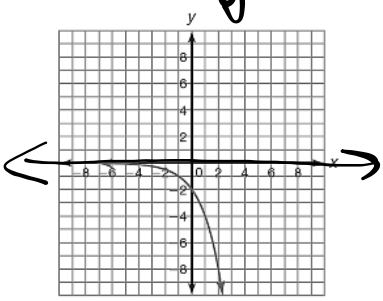
Maximum: none Minimum: none

Asymptote: y = 0



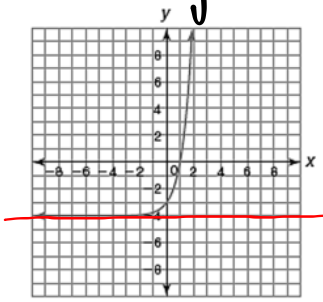
Maximum: none Minimum: none

Asymptote: y = 0



Maximum: none Minimum: none

Asymptote: y = 0



Maximum: none Minimum: none

Asymptote: y = -4

Algebra 1 Unit 10: Exponential Functions Notes

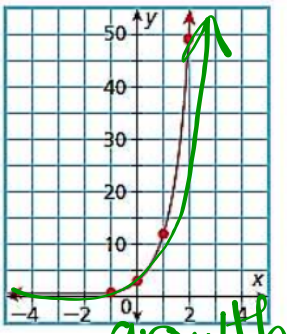
Intervals of Increase and Decrease

Interval of Increase		
Define: The part of the graph that is rising as you read left to right.	**Think:** From left to right, is my graph going up?	**Write:** An inequality using the x-value of the vertex
Interval of Decrease		
Define: The part of the graph that is falling as you read from left to right.	**Think:** From left to right, is my graph going down?	**Write:** An inequality using the x-value of the vertex

growth

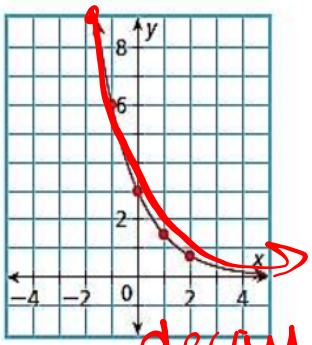
decay

LEFT (read) RIGHT



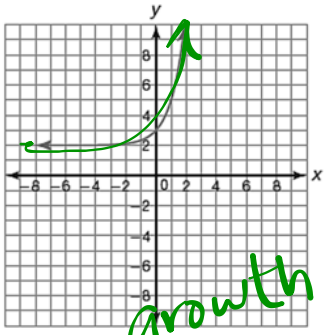
Interval of Increase: growth IR

Interval of Decrease: none



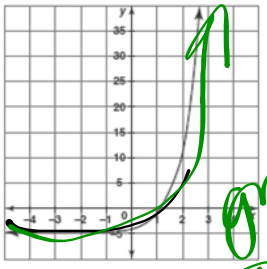
Interval of Increase: decay none

Interval of Decrease: IR



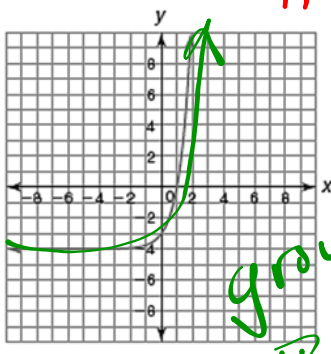
Interval of Increase: growth IR

Interval of Decrease: none



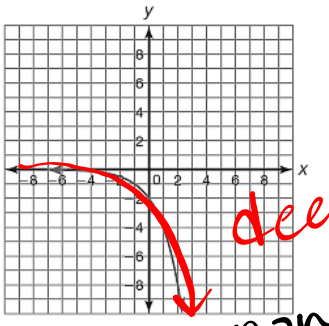
Interval of Increase: growth IR

Interval of Decrease: none



Interval of Increase: growth IR

Interval of Decrease: none



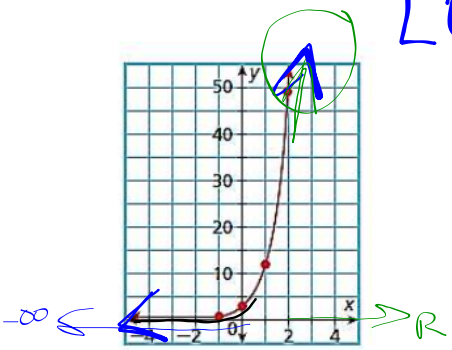
Interval of Increase: decay none

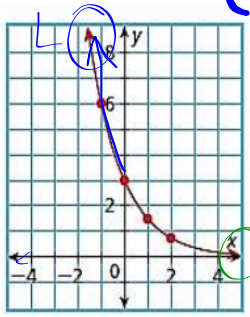
Interval of Decrease: IR

End Behavior

End Behavior	
Define: Behavior of the ends of the function (what happens to the y-values or f(x)) as x approaches positive or negative infinity. The arrows indicate the function goes on forever so we want to know where those ends go.	
Think: As x goes to the left (negative infinity), what direction does the left arrow go?	**Write:** As x → -∞, f(x) → _____
Think: As x goes to the right (positive infinity), what direction does the right arrow go?	**Write:** As x → ∞, f(x) → _____

Look @ arrows (→)



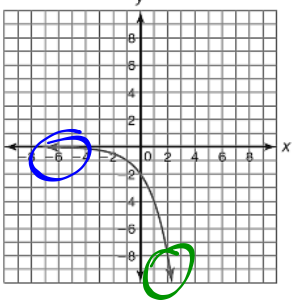


As x approaches -∞, f(x) approaches _____.

As x approaches ∞, f(x) approaches _____.

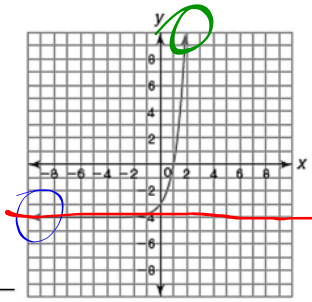
As x approaches -∞, f(x) approaches ∞.

As x approaches ∞, f(x) approaches 0.



As x approaches -∞, f(x) approaches _____.

As x approaches ∞, f(x) approaches _____.



y = -4

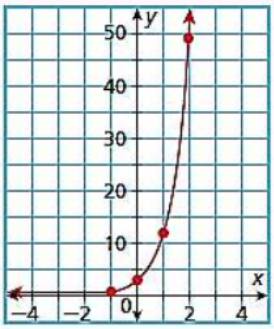
As x approaches -∞, f(x) approaches -4.

As x approaches ∞, f(x) approaches ∞.

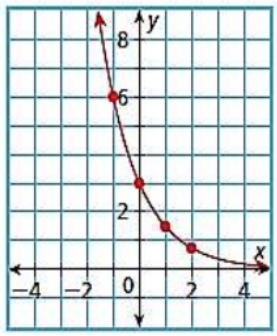
Algebra 1 Unit 10: Exponential Functions Notes

Average Rate of Change from a Graph

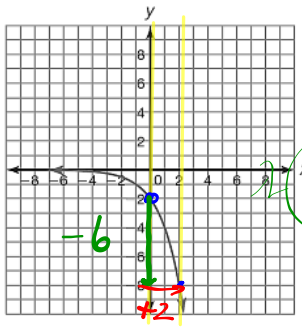
Average Rate of Change: Rate of change or slope for a given interval on a graph. The given interval is written using the inequality notation $a \le x \le b$, where a and b represent the initial and final x-value of the interval.



Calculate the average rate of change for the interval $0 \le x \le 2$



Calculate the average rate of change for the interval $-1 \le x \le 2$



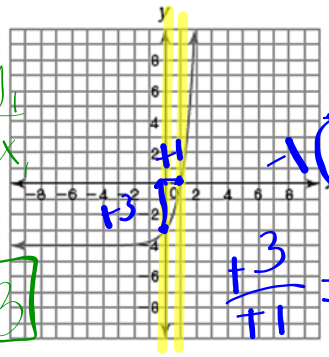
x	y
0	−2
2	−8

$\frac{y_2 - y_1}{x_2 - x_1}$ +6

$\frac{6}{2} = 3$

$\frac{-6}{2} = -3$

Calculate the average rate of change for the interval $0 \le x \le 2$



x	y
0	−3
1	0

$\frac{+3}{+1} = 3$ $\frac{-3}{-1} = 3$

Calculate the average rate of change for the interval $0 \le x \le 1$

Average Rate of Change from an Equation

If you are given an equation of a function and asked to calculate the average rate of change for that function over a given interval, you will substitute the initial x-value and the final x-value into the function to create two sets of ordered pairs. Then using the ordered pairs, substitute into the slope formula.

a. $y = 3^x$ $-1 \le x \le 3$

x	y
−1	$\frac{1}{3}$
3	27

$y = 3^{-1} = \frac{1}{3}$

$y = 3^3 = 27$

$\frac{-24}{-2} = 12$

b. $y = 2(1/2)^x$ $-4 \le x \le 0$

x	y
−4	32
0	2

ROC $= \frac{30}{-4} = \frac{15}{-2}$