Day 1 – Arithmetic & Geometric Sequences

For the following patterns, find the next two numbers. Then describe the rule you are applying each time.

Pattern	Rule	Classify
a. -4, -2, 0, 2, ______, ______ …	______________________	________
b. -20, -16, -12, -8, -4, ______, ______ …	______________________	________
c. 5, 25, 125, 625, ______, ______ …	______________________	________
d. 6.5, 5, 3.5, 2, ______, ______ …	______________________	________
e. 192, 96, 48, 24, ______, ______ …	______________________	________
f. 12, 18, 24, ______, ______ …	______________________	________
g. 81, 27, 9, 3, ______, ______ …	______________________	________
h. 50, 40, 30, ______, ______ …	______________________	________
i. 2, 8, 32, 128, ______, ______ …	______________________	________
j. 11, 9, 7, ______, ______ …	______________________	________
k. 64, -32, 16, -8, ______, ______ …	______________________	________
l. 75, 15, 3, ______, ______ …	______________________	________

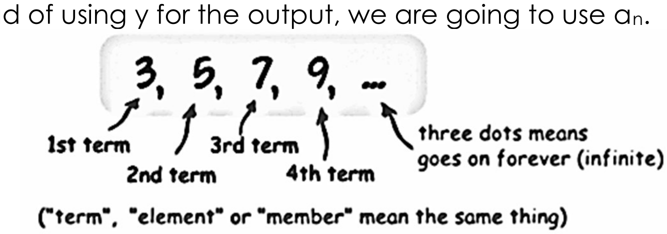
g. What did you notice about your patterns? ______________________________________

h. What do you think the "…" means? ______________________________________

Sequences

A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects.

What you may not realize is when it comes to sequences, they are considered a type of function. The position of each term is called the **term number or term position.** We can think of the term number or position as the input (domain) and the actual term in the sequence as the output (range). Instead of using x for the input, we are going to use n and instead of using y for the output, we are going to use a_n.



Pattern A:

Term Number (n)						
Term (a_n)	-4	-2	0	2		

Pattern K:

Term Number (n)					
Term (a_n)	81	27	9		

Types of Sequences

Information	Arithmetic	Geometric
Type of Function		
Created by…	Adding or Subtracting the same number each time. Called a ____________________	Multiplying by the same number each time. Called a ____________________
Explicit Formula (allows you to find nth term)	$a_n = a_1 + d(n-1)$ a_n: ____________________ a_1: ____________________ n: ____________________ d: ____________________	$a_n = a_1 * r^{n-1}$ a_n: ____________________ a_1: ____________________ n: ____________________ r: ____________________
Generating a Pattern	Find the first five terms: $a_n = 4 + 3(n-1)$	Find the first five terms: $a_n = 3 \cdot 5^{n-1}$
Converting to Function Form	Convert $a_n = 4 + (n-1)3$	Convert $a_n = 3 \cdot 5^{n-1}$

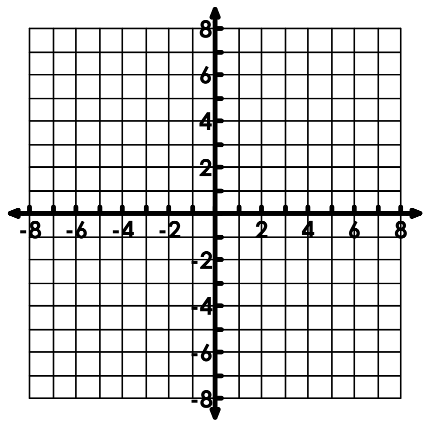
Graphing Sequences

For the following sequences, complete the following:

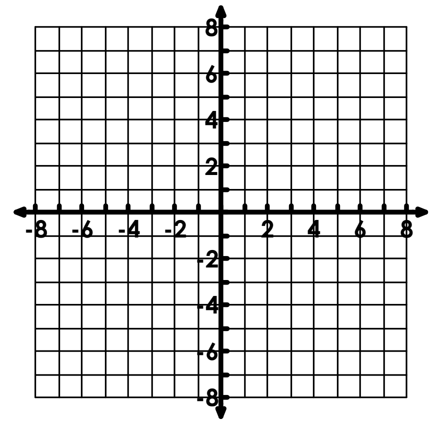
a. Create a table representing the term numbers and terms and then graph

b. Create an Explicit Rule to describe the sequence.

1. -8, -5, -2, 1 …



2. 4, 2, 1, .5 …



b. Explicit Rule:

b. Explicit Rule:

Generating a Sequence from an Explicit Formula

For the following sequences, find the first five terms:

a. $a_n = -3 + 2(n-1)$

b. $a_n = -3 \cdot 2^{n-1}$

c. $a_n = 9n + 2$

d. $a_n = 2 \cdot 4^{n-1}$

e. $a_n = (-3)^{n-1}$

f. $a_n = -(n-1)$

Why We Have a Formula for Sequences

Take a look at the following pattern: **4, 8, 12, 16 ….**

What is the 3rd term? _________ What is the 5th term? _________ What is the 7th term? ________

What is the pattern? ____ What is the 1st term? ________

What is the 54th term? ________ (You don't want to add ____ over and over 54 times?!?!?!?)

This is why the **Explicit Formula** was created – as long as you know your common difference and 1st term, you can create a rule to describe any arithmetic sequence and use it to find any term you want.

Finding the Nth Term

To find the nth term, particularly when the nth term is quite large, you want to create an Explicit Rule first and then substitute that term number into the rule for n. For the given sequences, create an explicit rule and then use the rule to find the following terms:

a. 10, 15, 20, 25, ….. Find 21st term

b. 121, 110, 99, 88 …. Find a_{16}

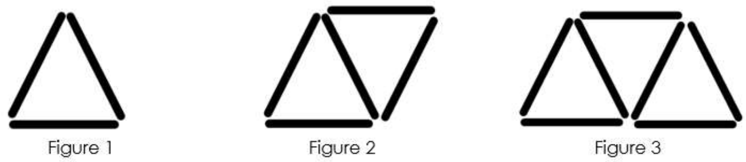
c. 1.5, 4.5, 13.5 ….. Find a_7

d. -30, -22, -14, -6 …. Find a_{30}

e. 162, 108, 72, 48 …. Find 8th term

f. 1, -2, 4, -8, … Find 10th term

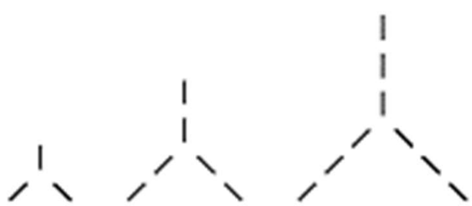
Using Figures to Create Rules



	# of Popsicle Sticks	Perimeter
Figure 1		
Figure 2		
Figure 3		
Figure 4		
Figure 5		
Figure 6		

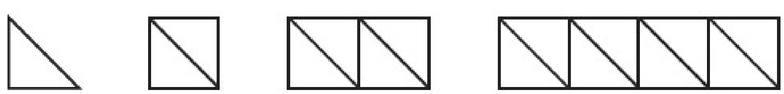
a. Create an explicit rule for finding the number of Popsicle sticks.

b. Create an explicit rule for finding the perimeter.



	# of Dashes
Figure 1	
Figure 2	
Figure 3	
Figure 4	
Figure 5	
Figure 6	

a. Create an explicit rule for finding the number of dashes.



	# of Triangles
Figure 1	
Figure 2	
Figure 3	
Figure 4	
Figure 5	

Figure 6	

a. Create an explicit rule for finding the number of triangles.

Day 2 – Recursive Formulas & More with Sequences

Information	Arithmetic	Geometric
Recursive Formula (allows you to find next term)	**a_1 = first number** **$a_n = a_{n-1} + d$** a_n: ____________ a_{n-1}: ____________ d: ____________	**a_1 = first number** **$a_n = r(a_{n-1})$** a_n: ____________ a_{n-1}: ____________ r: ____________

Generating a Sequence from a Recursive Formula

For each of the following recursive formulas, generate the first five terms.

a. $a_1 = 7$
$a_n = a_{n-1} + 4$

b. $a_1 = -54$
$a_n = \frac{1}{3}(a_{n-1})$

c. $a_1 = -3.5$
$a_n = a_{n-1} + 9$

d. $a_1 = 4$
$a_n = 2(a_{n-1})$

e. $a_1 = -7$
$a_n = a_{n-1} - 6$

f. $a_1 = 1025$
$a_n = \left(\frac{1}{5}\right)(a_{n-1})$

Creating Explicit and Recursive Formulas

For each of the following sequences, define the first term and common difference/constant ratio. Then create a simplified explicit formula and recursive formula.

a. 1, 8, 15 …	**b. 4, 0, -4 …**	**c. 400, 200, 100 …**
Type:	Type:	Type:
Explicit:	Explicit:	Explicit:
Recursive:	Recursive:	Recursive:
d. 3, 6, 12 …	**e. -5, 3, 11 …**	**f. 40, 10, $\frac{5}{2}$ …**
Type:	Type:	Type:
Explicit:	Explicit:	Explicit:

Recursive:	Recursive:	Recursive:

Challenge

a. Two terms of an arithmetic sequence are $a_5 = 15$ and $a_6 = 22$.

a. What is the common difference?

b. What are the first four terms of this sequence?

c. Write the **EXPLICIT** and **RECURSIVE** rules for this sequence.

b. Two terms of a geometric sequence are $a_5 = 162$ and $a_6 = 486$.

a. What is the constant ratio?

b. What are the first four terms of this sequence?

c. Write the **EXPLICIT** and **RECURSIVE** rules for this sequence.

c. Given $a_{10} = 16$ and $d = 5$, write the **EXPLICIT** and **RECURSIVE** rules for this sequence.

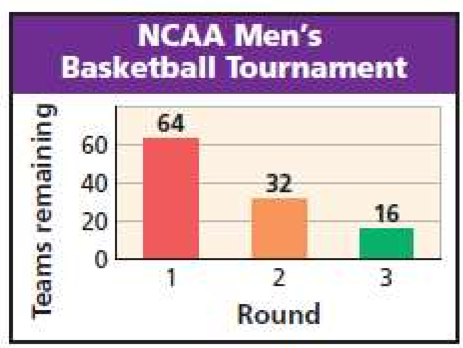
Day 3 – Comparing Arithmetic & Geometric Sequences

Now it's time to apply arithmetic and geometric sequences to real world contexts.

Arithmetic	Geometric
Add or Subtract by the same number (common difference)	Multiply by the same number (constant ratio)
Explicit: $a_n = a_1 + (n - 1)d$	Explicit: $a_n = a_1 \cdot r^{n-1}$
Recursive: $a_n = a_{n-1} + d$	Recursive: $a_n = r(a_{n-1})$

For each of the following problems, determine if it is arithmetic or geometric, create an explicit rule, and then answer the question:

1. In the NCAA men's basketball tournament, 64 teams compete in round 1. Fewer teams remain in each following round, as shown in the graph. How many teams compete in Round 6?



Type: ___________________

Explicit Formula: ________________________

Solution: ________________________________

2. The odometer on a car reads 60,473 on Day 1. Every day, the car is driven 54 miles. If this pattern continues, what is the odometer reading on Day 20?

Type: ___________________

Explicit Formula: ______________________

Solution: _____________________________

3. To package and ship an item, it costs $5.75 for the first pound and $0.75 for each additional pound. What is the cost of shipping of 12 pound package?

Type: _________________

Explicit Formula: ______________________

Solution: _____________________________

4. The table shows a car's value for 3 years after it is purchased. How much will the car be worth in the 10th year?

Type: _________________

Explicit Formula: ______________________

Solution: _____________________________

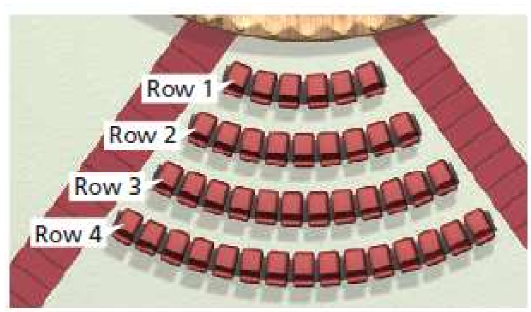
Year	Value ($)
1	10,000
2	8,000
3	6,400

5. Seats in a concert hall are arranged in the pattern shown. How many seats are in the 15th row?

Type: _________________

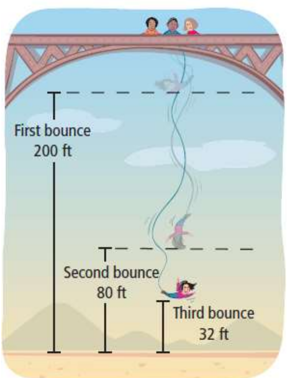
Explicit Formula: ______________________

Solution: _____________________________



b. A ticket costs $40. Suppose every seat in the first 10 rows is filled. What is the total revenue from those seats?

6. A bungee jumper jumps from a bridge. The diagram shows the bungee jumper's height above the ground at the top of each bounce. What is the bungee jumper's height at the top of the 5th bounce?



Type: ___________________

Explicit Formula: ________________________

Solution: ________________________________

7. Three years ago, the annual tuition at a university was $3000. The tuition for the next few years can be modeled in the table to the right. Let the year 2016 represent year 1.

Type: ___________________

Explicit Formula: ________________________

a. How much was the tuition in 2013? __________________

b. How much will the tuition be in 2020? __________________

Year	Tuition
2016	$3000
2017	$3300
2018	$3630

8. Karen started selling bagels to offices in her area. Her sales for the first three months are shown in the table. If this trend continues, find the amount of sales in Month 8.

Type: ________________

Explicit Formula: ______________________

Solution: _____________________________

Month	Sales ($)
1	$200.00
2	$230.00
3	$264.50