Algebra 1 Unit 11: Comparing Linear, Quadratic, & Exponential Functions Notes

Day 1 – Arithmetic & Geometric Sequences

add/sub mult/div

For the following patterns, find the next two numbers. Then describe the rule you are applying each time.

Pattern	Rule	Classify
a. -4, -2, 0, 2, 4, 6 ...	add 2	a
b. -20, -16, -12, -8, -4, 0, 4 ...	+4	a
c. 5, 25, 125, 625, 3125, 15625 ...	mult by 5	g
d. 6.5, 5, 3.5, 2, 0.5, -1.5 ...	-1.5	a
e. 192, 96, 48, 24, 12, 6 ...	÷2	g
f. 12, 18, 24, 30, 36 ...	+6	a
g. 81, 27, 9, 3, 1, 1/3 ...	÷3	g
h. 50, 40, 30, 20, 10...	-10	a
i. 2, 8, 32, 128, 512, 2048...	×4	g
j. 11, 9, 7, 5, 3 ...	-2	a
k. 64, -32, 16, -8, 4, -2 ...	÷ -2	g
l. 75, 15, 3, 0.6, 0.12...	÷ 5	g

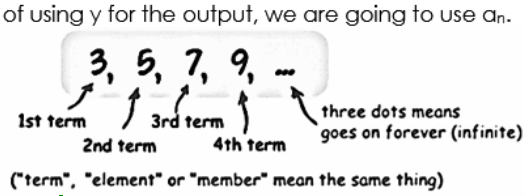
g. What did you notice about your patterns? in order

h. What do you think the "…" means? Keeps going

Sequences

A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects.

What you may not realize is when it comes to sequences, they are considered a type of function. The position of each term is called the **term number or term position**. We can think of the term number or position as the input (domain) and the actual term in the sequence as the output (range). Instead of using x for the input, we are going to use n and instead of using y for the output, we are going to use a_n.



Pattern A: arithmetic

x	Term Number (n)	1	2	3	4	5	6
y	Term (a_n)	-4	-2	0	2	4	6

+2 +2 +2

Pattern K: geometric

x	Term Number (n)	1	2	3	4	5
y	Term (a_n)	81	27	9	3	1

÷3 ÷3 ÷3

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Types of Sequences

Information	Arithmetic	Geometric
Type of Function	linear	exponential
Created by...	Adding or Subtracting the same number each time. Called a common difference	Multiplying by the same number each time. Called a constant ratio
Explicit Formula (allows you to find nth term)	$a_n = a_1 + d(n-1)$ a_n: term a_1: 1st term (#) n: nth term d: difference	$a_n = a_1 * r^{n-1}$ a_n: term a_1: 1st term (#) n: nth term r: ratio
Generating a Pattern	Find the first five terms: $a_n = 4 + 3(n-1)$ n | $4+3(n-1)$ | a_n 1 | $4+3(1-1)$ | 4)+3 2 | $4+3(2-1)$ | 7)+3 3 | $4+3(3-1)$ | 10)+3 4 | $4+3(4-1)$ | 13	Find the first five terms: $a_n = 3\cdot 5^{n-1}$ n | $3\cdot 5^{n-1}$ | a_n 1 | $3\cdot 5^{1-1}$ | 3)·5 2 | | 15)·5 3 | | 75)·5 4 | | 375
Converting to Function Form	Convert $a_n = 4 + (n-1)3$	Convert $a_n = 3\cdot 5^{n-1}$

2

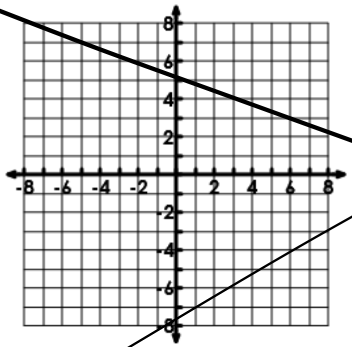
Graphing Sequences

For the following sequences, complete the following:

a. Create a table representing the term numbers and terms and then graph

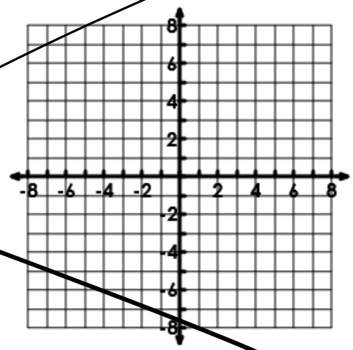
b. Create an Explicit Rule to describe the sequence.

1. -8, -5, -2, 1 …



b. Explicit Rule:

2. 4, 2, 1, .5 …



b. Explicit Rule:

Generating a Sequence from an Explicit Formula

For the following sequences, find the first five terms:

a. $a_n = -3 + 2(n-1)$

n	a_n
1	-3
2	-1
3	1
4	3

+2

b. $a_n = -3(2)^{n-1}$

n	a_n
1	-3
2	-6
3	-12
4	-24

·2

·2

c. $a_n = 9n + 2$

d. $a_n = 2 \cdot 4^{n-1}$

e. $a_n = (-3)^{n-1}$

f. $a_n = -(n-1)$

Why We Have a Formula for Sequences

Take a look at the following pattern: 4, 8, 12, 16, … 20 24 28

1 2 3 4 5 6 7

+4

What is the 3rd term? 12 What is the 5th term? 20 What is the 7th term? 28

What is the pattern? +4 What is the 1st term? 4

What is the 54th term? ? (You don't want to add 4 over and over 54 times?!?!?!?)

This is why the **Explicit Formula** was created – as long as you know your common difference and 1st term, you can create a rule to describe any arithmetic sequence and use it to find any term you want.

Finding the Nth Term

To find the nth term, particularly when the nth term is quite large, you want to create an Explicit Rule first and then substitute that term number into the rule for n. For the given sequences, create an explicit rule and then use the rule to find the following terms:

a. 10, 15, 20, 25, ….. Find 21st term

1 2 3 4

+5

$\boxed{a_n} = a_1 + d(n-1)$

$d = 5$

$a_1 = 10$

$a_{21} = 10 + 5(21-1) = 110$

b. 121, 110, 99, 88 …. Find a_{16}

1 2 3 4

−11 −11

$d = -11$

$a_1 = 121$

$a_{16} = 121 + -11(16-1)$

$a_{16} = -44$

c. 1.5, 4.5, 13.5 ….. Find a_7

·3 ·3

$r = 3$

$a_1 = 1.5$

$a_n = a_1 \cdot r^{n-1}$

$a_7 = 1.5 \cdot 3^{(7-1)}$

$a_7 = 1093.5$

d. -30, -22, -14, -6 …. Find a_{30}

+8 +8

$d: +8$

$a_1: -30$

$a_{30} = -30 + 8(30-1)$

$a_{30} = 202$

e. 162, 108, 72, 48 …. Find 8th term

$\times \frac{2}{3}$ $\times \frac{2}{3}$

TIP: $\frac{108}{162} = \frac{2}{3}$

$r: \frac{2}{3}$

$a_1 = 162$

$a_8 = 162 \cdot (\frac{2}{3})^{(8-1)}$

$a_8 = 9.48$

f. 1, -2, 4, -8, … Find 10th term

1

·−2 ·−2

$r: -2$

$a_1 = 1$

$a_n = a_1 \cdot r^{n-1}$

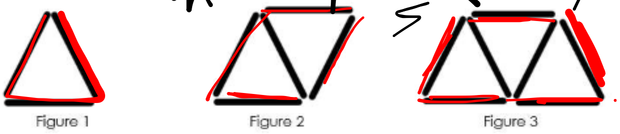
$a_{10} = 1 \cdot (-2)^{(10-1)}$

$a_{10} = -512$

Using Figures to Create Rules

outside

$a_n = a_1 + d(n-1)$



	# of Popsicle Sticks	Perimeter
Figure 1	3	3
Figure 2	5	4
Figure 3	7	5
Figure 4	9	6
Figure 5	11	7
Figure 6	13	8

+2 +2 +1 +1

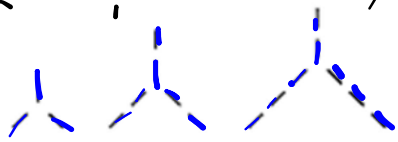
a. Create an explicit rule for finding the number of Popsicle sticks.

$a_n = 3 + 2(n-1)$

b. Create an explicit rule for finding the perimeter.

$a_n = 3 + 1(n-1)$

$a_n = a_1 + d(n-1)$



	# of Dashes
Figure 1	3
Figure 2	6
Figure 3	9
Figure 4	
Figure 5	
Figure 6	

+3 +3

a. Create an explicit rule for finding the number of dashes.

$a_n = 3 + 3(n-1)$

$r = \cdot 2$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 1 \cdot 2^{n-1}$

	# of Triangles
Figure 1	1
Figure 2	2
Figure 3	4
Figure 4	8
Figure 5	16

·2 ·2 ·2 ·2