Foundations of Algebra | Unit 4: Equations & Inequalities | Notes

Day 3 –Justifying the Solving of Equations

Properties of Addition Operations			
Property	**What It Means**	**General Example**	**Example 1**
Commutative Property of Addition	Rearrange the order and the sum will stay the same.	$a + b = b + a$	$2 + 4 = 4 + 2$
Associative Property of Addition	Change the order of the grouping and the sum will stay the same.	$(a + b) + c = a + (b + c)$	$(4 + 6) + 1 = 4 + (6 + 1)$
Additive Identity	Zero added to any number will equal that number.	$a + 0 = a$ $\# + 0 = \#$	$4 + 0 = 4$
Additive Inverse	A number plus its inverse equals 0.	$a + -a = 0$	$7 + -7 = 0$
Properties of Multiplication Operations			
Commutative Property of Multiplication	Rearrange the order and the product will stay the same.	$a \cdot b = b \cdot a$	$5 \cdot 2 = 2 \cdot 5$ $10 = 10$
Associative Property of Multiplication	Change the order of the grouping and the product will stay the same.	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(3 \cdot 4) \cdot 2 = 3 \cdot (4 \cdot 2)$
Multiplicative Identity	One times any number equals that number.	$a \cdot 1 = a$	$8 \cdot 1 = 8$
Multiplicative Inverse (Reciprocal)	A number times its reciprocal equals 1.	$a \cdot \frac{1}{a} = 1$ $\frac{a}{1} \cdot \frac{1}{a} = \frac{a}{a} = 1$	$3 \cdot \frac{1}{3} = 1$ $\frac{3}{1} \cdot \frac{1}{3} = \frac{3}{3} = 1$
Zero Property of Multiplication	Any number times 0 will always equal 0.	$a \cdot 0 = 0$	$7 \cdot 0 = 0$
Distributive Property distribute	Multiply a number to every term within a quantity (parenthesis).	$a(b + c) = ab + ac$	$4(x + 5) = 4x + 4(5)$ $= 4x + 20$

Practice: Each of the following expressions has been simplified one step at a time. Next to each step, identify the property or simplification used in the step.

1. $4 + 5(x + 7)$ **Given**
   $4 + (5x + 35)$ _____
   $5x + 4 + 35$ _____
   $5x + (4 + 35)$ _____
   $5x + 39$ _____

2. $4(10x + 2) - 40x$ **Given**
   $40x + 8 - 40x$ _____
   $8 + 40x - 40x$ _____
   $8 + 0$ _____
   8 _____

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Justifying the Solving of Equations

Properties of Equality		
Property	**General Example**	**Example 1**
+ **Addition Property of Equality**	If a = b, then a + c = b + c	If x – 4 = 8, then x = 12 (+4 +4)
— **Subtraction Property of Equality**	If a = b, then a – c = b - c	If x + 5 = 7, then x = 2 (-5 -5)
· **Multiplication Property of Equality**	If a = b, then ac = bc	2· If $\frac{x}{2} = 9$ then x = 18 (x = 18)
÷ **Division Property of Equality**	If a = b, then $\frac{a}{c} = \frac{b}{c}$	If $\frac{2x}{2} = \frac{10}{2}$, then x = 5
Reflexive Property ("mirror" or reflection)	a = a	5 = 5
Symmetric Property	If a = b, then b = a	If 2 = x, then x = 2
Transitive Property ("middle man") (a=b, b=c, a=c)	If a = b and b = c, then a = c	If x + 2 = y and y = 4x + 3, then x + 2 = 4x + 3
Substitution Property	If x = y, then y can be substituted for x in any expression	If x = 3 and the expression is 2x – 7, then 2(3) - 7

Practice: Using properties of operations and equality, list each property next to each step in the equation solving process. *READ BTWN THE LINES*

Example 1

· x + 4 = 9 (-4 -4)	**Given**
x = 5	sub prop of equality

Example 2

+5 7 = x - 5 +5	**Given**
12 = x	add prop of equality
x = 12	Symmetric prop

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Example 3

$3 \cdot \frac{x}{3} = 5 \cdot 3$	**Given**
x = 15	multiply prop of equality

Example 4

$\frac{6x}{6} = \frac{24}{6}$	**Given**
x = 4	division prop of equality

Justifying the Solutions to Two & Multi-Step Equations

Practice: Identify the property or simplification that is used in each step to solve the equation.

Example 1

$3x + 5 = -13$ $-5 \quad -5$	**Given**
$\frac{3x}{3} = \frac{-18}{3}$	sub property of equality
x = -6	division property of equality

Example 2

12 = 2(x – 4)	**Given**
12 = 2x – 8 $+8 \quad +8$	distributive prop.
$\frac{20}{2} = \frac{2x}{2}$	add prop of equality
10 = x	division prop of equality
x = 10	Symmetric prop

Example 3

5n – 3 = 2(n + 3) + 9	**Given**
5n – 3 = 2n + 6 + 9	distributive prop.
5n – 3 = 2n + 15 $-2n \quad -2n$	"combine like terms" CLT
3n – 3 = 15 $+3 \quad +3$	sub prop of equality
$\frac{3n}{3} = \frac{18}{3}$	add prop of equality
n = 6	division prop of equality

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Special Types of Solutions

Solve the following equations. What do you notice about the solutions?

a. $2x - 7 + 3x = 4x + 2$

$5x - 7 = 4x + 2$

$-4x \quad -4x$

$x - 7 = 2$

$+7 \quad +7$

$x = 9$

(one solution)

b. $3(x - 5) + 11 = x + 2(x + 5)$

$3x - 15 + 11 = x + 2x + 10$

$3x - 4 = 3x + 10$

$-3x \quad -3x$

$-4 \neq 10$

NO SOLUTION!

c. $3x + 7 = 5x + 6 - 2x + 1$

$3x + 7 = 3x + 7$

$+7 \quad -7$

$\frac{3x}{3} = \frac{3x}{3}$

$x = x$

Infinite Solution

We are going to use the graphing calculators to view what these equations look like. Draw a sketch of the graphs:

Problem A: Problem B: Problem C:

Conclusions: