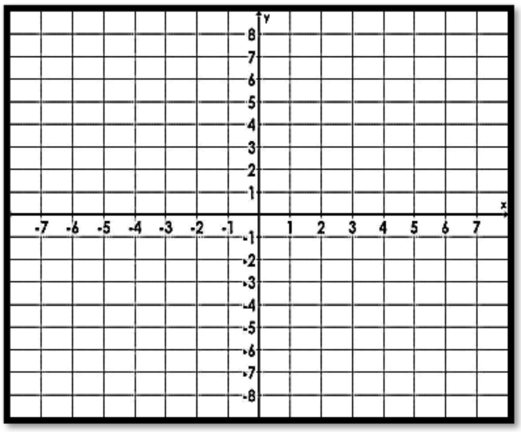
Day 1: Quadratic Transformations (H & K values)

The **parent function** of a function is the simplest form of a function. The parent function for a quadratic function is $y = x^2$ or $f(x) = x^2$. Graph the parent function below.

x	x^2
-3	
-2	
-1	
0	
1	
2	
3	



As you can see, the graph of a quadratic function is very different than the graph of a linear function.

The U-shaped graph of a quadratic function is called a ____________________.

The highest or lowest point on a parabola is called the ____________________.

One other characteristic of a quadratic equation is that one of the terms is always ________________.

There are several different forms a quadratic function can be written in, but the one we are going to work with for today is called **vertex form**. In the following explorations below, you are going to learn the effect of a, h, and k values have on the parent graph.

Vertex Form

$$f(x) = a(x-h)^2 + k$$

Variable	Summary of the Effects of the Transformations			
a	reflect −	Up: $a > 0$	width	skinny Stretch: $a > 1$
		Down: $a < 0$		Shrink: $a < 1$ wide
h	shifts left/right		Left: $h = \overline{\text{neg}}$	looks like $y=(x+h)^2$
			Right: $h = \overset{+}{\text{pos}}$	$y=(x-h)^2$
k	shifts ↑/↓		Up: $k = +$	
			Down: $k = -$	

Vertex: (h, k)

x-coord y-coord.

Slide 5 ~ The A Value, part 1 ~ $y = ax^2$

a. What does the a value do the blue graph? ______

b. When a is greater than 1, what does it do to the blue graph? ______

c. When a is between 0 and 1, what does it do to the blue graph? ______

d. If there is only an a value, what will the vertex always be? ______

Slide 6 ~ The A Value, part 2 ~ $y = ax^2$

a. What does the a value do the blue graph? ______

b. When a is less than 1, what does it do to the blue graph? ______

Practice: Describe the transformations from the given function to the transformed function. $y = ax^2$

a. $f(x) = x^2 \rightarrow f(x) = 4x^2$
$a = 4$
stretches by 4

b. $y = x^2 \rightarrow y = \frac{1}{4}x^2$
$a = \frac{1}{4}$
shrink by $\frac{1}{4}$

c. $f(x) \rightarrow 6f(x)$
$a = 6$
stretch by 6

d. $f(x) = x^2 \rightarrow f(x) = -x^2$
$a = -1$
reflects

f. $y = x^2 \rightarrow y = -\frac{1}{2}x^2$
$a = -\frac{1}{2}$
Shrink by $\frac{1}{2}$; reflect

g. $f(x) \rightarrow -4f(x)$
$a = -4$
reflect
stretch by 4

Putting It All Together with A, H, and K

Practice: Given the equations below, name the vertex and describe the transformations:

Equation	Transformations	Vertex
1. $y = -(x-4)^2 + 7$	$a = -1 \rightarrow$ reflects $h = +4 \rightarrow$ right 4 $k = +7 \rightarrow$ up 7	(4, 7)
2. $y = -2(x+2)^2 + 5$	$a = -2 \rightarrow$ reflect, stretch by 2 $h = -2 \rightarrow$ left 2 $k = +5 \rightarrow$ up 5	(-2, 5)
3. $y = \frac{1}{2}(x-3)^2 - 8$	$a = \frac{1}{2} \rightarrow$ shrink by $\frac{1}{2}$ $h = +3 \rightarrow$ right 3 $k = -8 \rightarrow$ down 8	(3, -8)

Practice: Create an equation to represents the following transformations: $y = a(x-h)^2 + k$

a. Shifted down 4 units, right 1 unit, and reflected across the x-axis
$k = -4$ $h = +1$ $a = -1$ $y = -(x - 1)^2 - 4$

b. Shifted up 6 units, reflected across the x-axis, and stretch by a factor of 3
$k = +6$ $a = -3$ $y = -3(x)^2 + 6$

c. Shifted up 2 units, left 4 units, reflected across the x-axis, and shrunk by a factor of ¾
$k = +2$ $h = -4$ $a = -\frac{3}{4}$ $y = -\frac{3}{4}(x+4)^2 + 2$

Day 2 - Characteristics of Quadratics

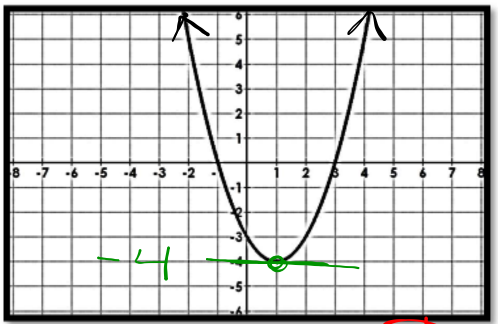
One key component to fully understanding quadratic functions is to be able to describe characteristics of the graph and its equation.

Domain and Range

Domain		
Define: All possible values of x	**Think:** How far left to right does the graph go?	**Write:** Smallest x ≤ x ≤ Biggest x *use < if the circles are open*

Range		
Define: All possible values of y	**Think:** How far down to how far up does the graph go?	**Write:** y ≤ highest y value (opens down) y ≥ lowest y value (opens up)

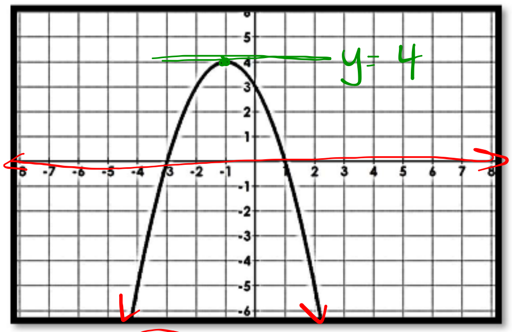
Graph 1



Domain: $-\infty < x < \infty$ $\mathbb{R}$

Range: $y \geq -4$

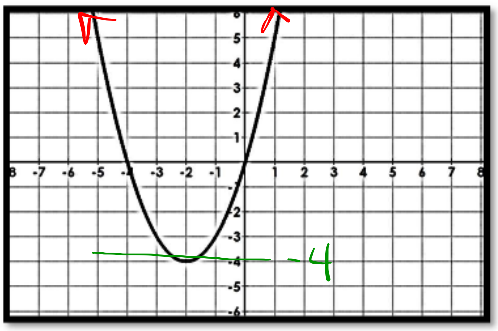
Graph 2



Domain: $\mathbb{R}$

Range: $y \leq 4$

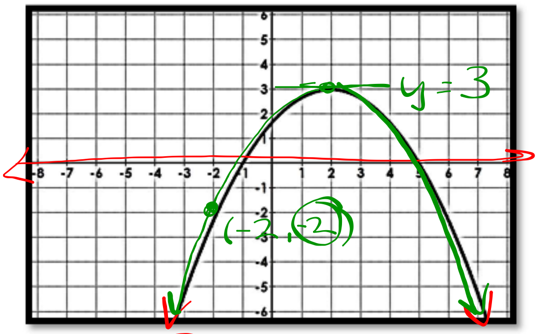
Graph 3



Domain: $\mathbb{R}$

Range: $y \geq -4$

Graph 4



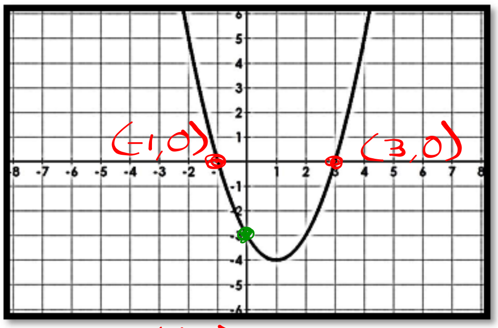
Domain: $\mathbb{R}$

Range: $y \leq 3$

Zeros and Intercepts

Y-Intercept		
Define: Point where the graph crosses the y-axis	**Think:** At what coordinate point does the graph cross the y-axis?	**Write:** (0, b)
X-Intercept		
Define: Point where the graph crosses the x-axis	**Think:** At what coordinate point does the graph cross the x-axis?	**Write:** (a, 0)
Zero		
Define: Where the function (y-value) equals 0	**Think:** At what x-value does the graph cross the x-axis?	**Write:** x = a

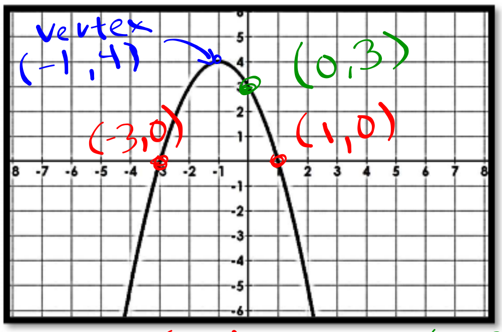
Graph 1



X-intercepts: (-1,0) (3,0) Y-intercept: (0,-3)

Zeros: $x = -1, 3$

Graph 2

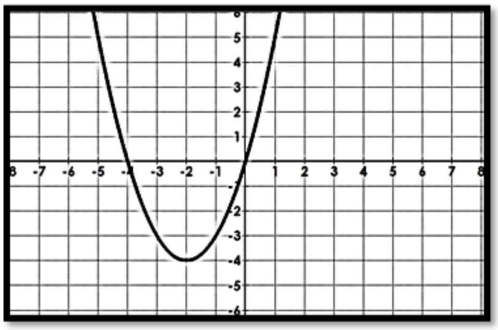


X-intercepts: (-3,0) (1,0) Y-intercept: (0,3)

Zeros: $x = -3, 1$

$y = -(x+1)^2 + 4$

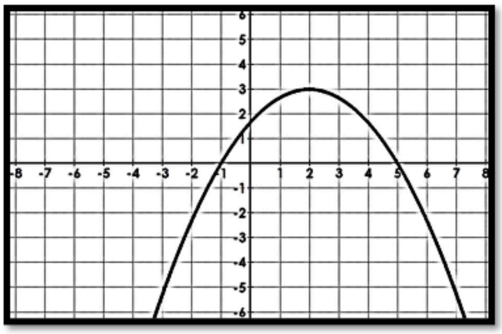
Graph 3



X-intercepts: Y-intercept:

Zeros:

Graph 4



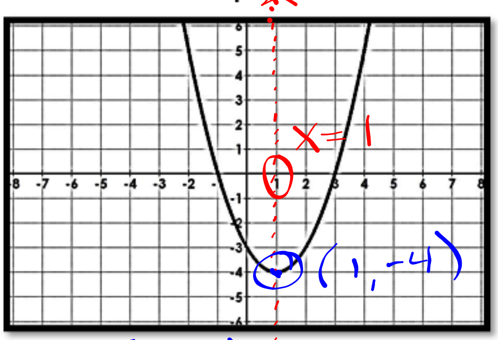
X-intercepts: Y-intercept:

Zeros:

Vertex & Axis of Symmetry

Vertex		
Define: Highest or lowest point or peak of a parabola	**Think:** What is my highest or lowest point on my graph?	**Write:** Name the point (h, k)
Axis of Symmetry		
Define: The vertical line that divides the parabola into mirror images and runs through the vertex	**Think:** What imaginary, vertical line would make the parabola symmetrical?	**Write:** x = h (x value of the vertex)

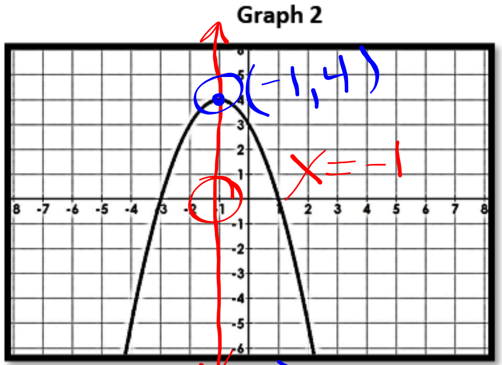
Graph 1



Vertex: (1, -4)

Axis of Symmetry: x = 1

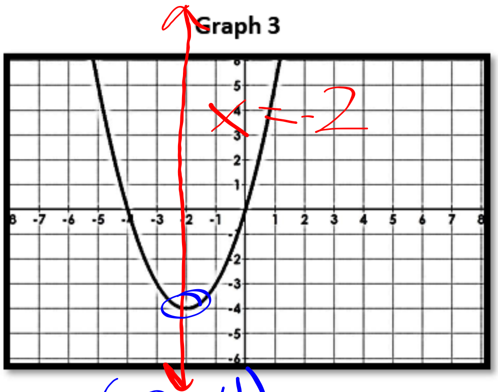
Graph 2



Vertex: (-1, 4)

Axis of Symmetry: x = -1

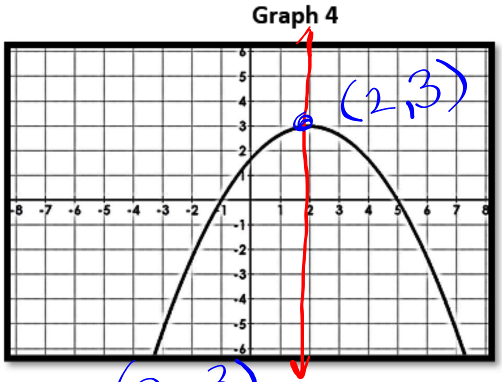
Graph 3



Vertex: (-2, -4)

Axis of Symmetry: x = -2

Graph 4



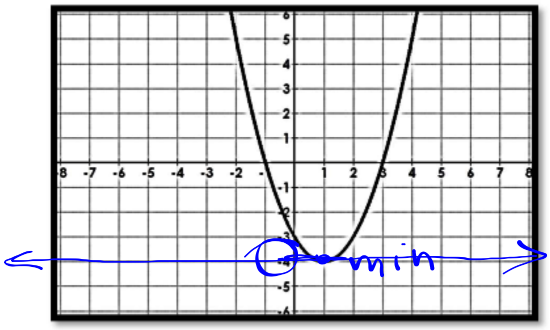
Vertex: (2, 3)

Axis of Symmetry: x = 2

Extrema

Maximum		
Define: Highest point or peak of a function.	**Think:** What is my highest point on my graph?	**Write:** y = k (y-value of the vertex)
Minimum		
Define: Lowest point or valley of a function.	**Think:** What is the lowest point on my graph?	**Write:** y = k (y-value of the vertex)

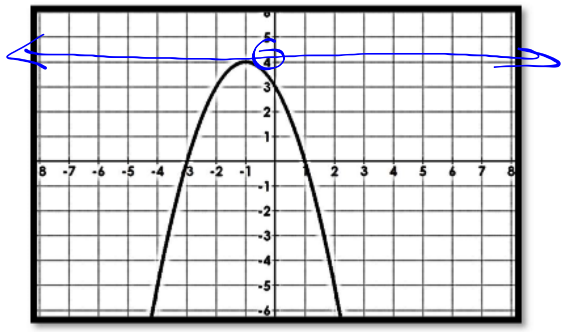
Graph 1



Extrema: min

Min/Max Value: y = -4

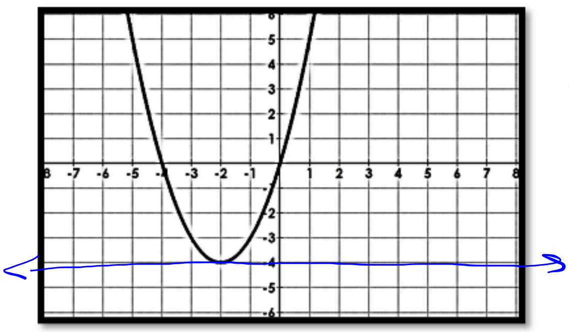
Graph 2



Extrema: max

Min/Max Value: y = 4

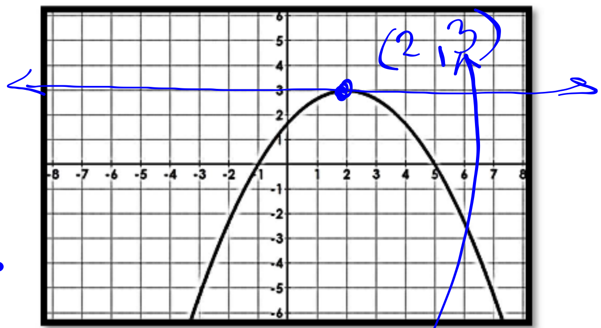
Graph 3



Extrema: min

Min/Max Value: y = -4

Graph 4



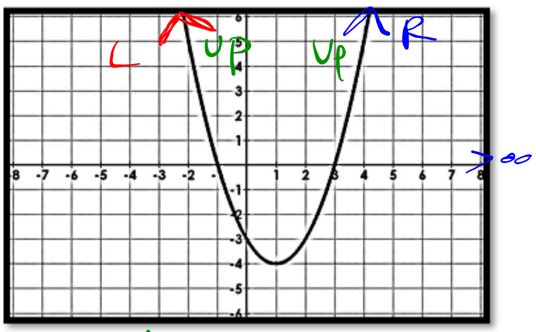
Extrema: max

Min/Max Value: y = 3

End Behavior

End Behavior	
Define: Behavior of the ends of the function (what happens to the y-values or f(x)) as x approaches positive or negative infinity. The arrows indicate the function goes on forever so we want to know where those ends go.	
Think: As x goes to the left (negative infinity), what direction does the left arrow go? L = -∞ , R = +∞	**Write:** As x → -∞, f(x) → _____
Think: As x goes to the right (positive infinity), what direction does the right arrow go? R = +∞	**Write:** As x → ∞, f(x) → _____

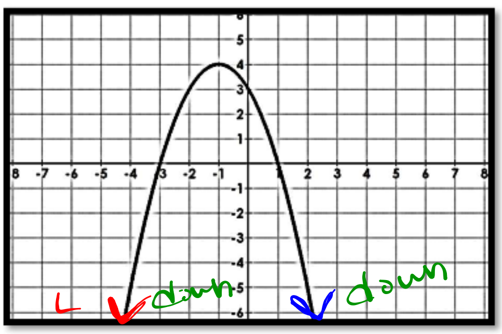
Graph 1



As x → -∞, f(x) → +∞.
y
L

As x → ∞, f(x) → +∞.
R

Graph 2

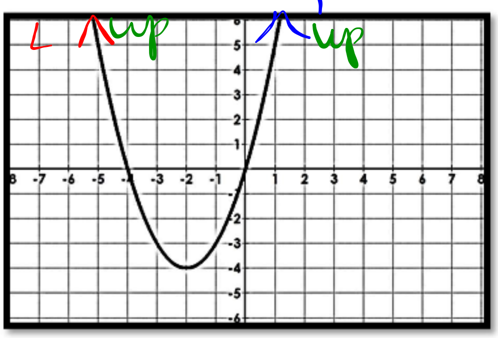


As x → -∞, f(x) → -∞.
L

As x → ∞, f(x) → -∞.
R

Graph 3

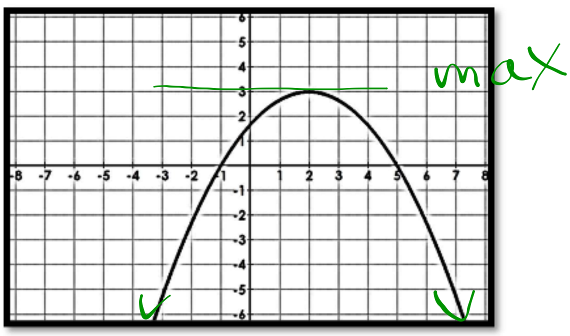
R



As x → -∞, f(x) → +∞.
L

As x → ∞, f(x) → +∞.
R

Graph 4



As x → -∞, f(x) → -∞.

As x → ∞, f(x) → -∞.