Unit 8: Quadratic Functions

Learning Goal 8.3 – Applications of Quadratic Functions

After completion of this unit, you will be able to…

- Calculate average rate of change from a graph, table, or equation.
- Calculate the vertex from a word problem and use it to answer real world questions
- Compare quadratic functions using multiple representations

Timeline for Unit 8

Monday	Tuesday	Wednesday	Thursday	Friday
20 ***No School***	**21** Day 1 – Transformations of Quadratic Functions	**22** Day 2 – Characteristics of Quadratic Functions	**23** Day 3 – Characteristics of Quadratic Functions	**24** Day 4 – **8.1 Learning Assessment**
27 Day 5 – Graphing in Vertex Form Graphing in Standard Form	**28** Day 6 – Graphing in Factored Form Practice	**29** Day 7 – Writing Equations of Parabolas	**30** Day 8 – Comparing Different Forms of Quadratics	**31** Day 9 – **8.2 Learning Assessment**
3 Day 10 – Average Rate of Change	**4** Day 11 – Applications of the Vertex	**5** Day 12 – Comparing Different Quadratic Functions	**6** Day 13 – Comparing Different Quadratic Functions	**7** Day 14 – **8.3 Learning Assessment**

Tutoring Times

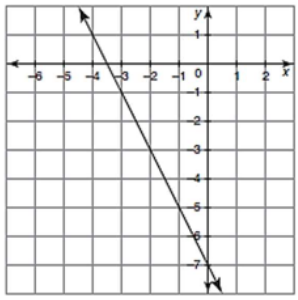
	Monday	Tuesday	Wednesday	Thursday	Friday
AM	Mrs. Jackson 7:45 – 8:15 Room 1210	Mr. Phillips 7:45 – 8:15 Room 1206	Mrs. Jackson & Mr. Webb 7:45 – 8:15 Room 1210 Room 1205	Mr. Watson & Mr. Phillips 7:45 – 8:15 Room 1208 Room 1206	Mr. Watson 7:45 – 8:15 Room 1208
PM	NONE	Mrs. Petersen 3:30 – 4:30 Room 1210	NONE	NONE	NONE

1

Day 10: Average Rate of Change

Review: Find the slope (average rate of change) for the following problems:

a.



b.

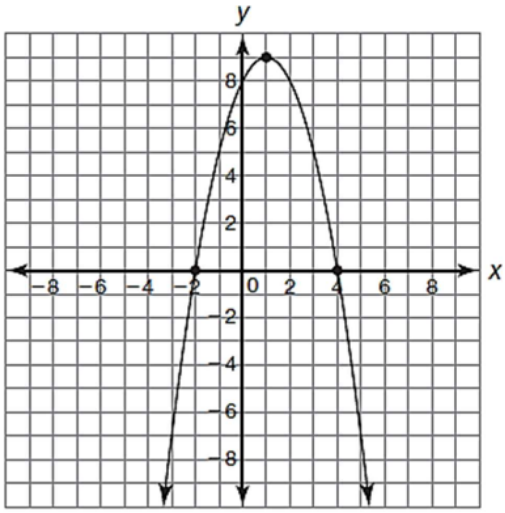
x	y
3	27
5	45
7	63
9	81

c. (-9, 5) & (-3, 1)

When you calculate the slope of linear function, its slope is ALWAYS _____.

Investigating the "Slope" of a Quadratic Function

The graph of $y = -x^2 + 2x + 8$ is given. Fill in the table of values on the right. Then determine the slope from one point to the next point.



X	Y
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

What do you notice about the rate of change as you go from one point to the next?

What do you notice if you find the difference of all the slopes?

Algebra 1 Unit 8: Quadratic Functions Notes

First versus Second Differences

Quadratic Functions have **constant second differences.** Second differences can be calculated by finding the rate of change with the first differences. Linear functions have **constant first differences.** Since quadratic functions do not have constant first differences, they do not have a slope that remains constant for the entire graph of a parabola.

a. $y = 2x$ _____

x	y	First Differences	Second Differences
−3			
−2			
−1			
0			
1			
2			
3			

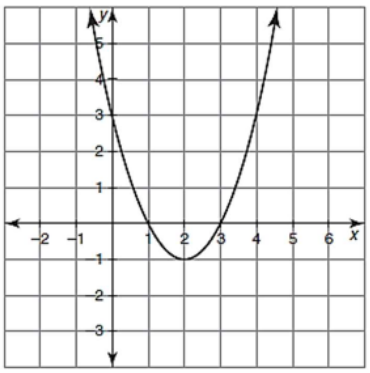
b. $y = 2x^2$ _____

x	y	First Differences	Second Differences
−3			
−2			
−1			
0			
1			
2			
3			

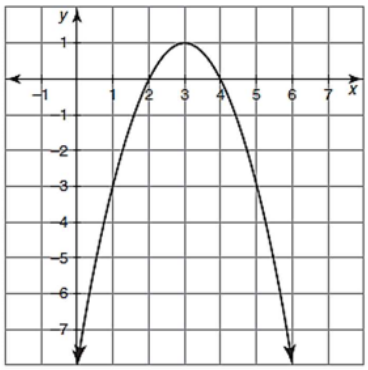
Therefore, you are never asked to find the slope of a quadratic function, but rather the **average rate of change** on a given interval. The average rate of change of a quadratic function will be different for each interval you are asked to find, just like in your investigation problem.

Practice: For the problems below, find the average rate of change for the given intervals:

Calculate average rate of change on interval **0 ≤ x ≤ 2**.



Calculate average rate of change on interval **0 ≤ x ≤ 3**.



Average Rate of Change without a Graph

If you are asked to calculate the average rate of change on an interval without a graph, you will have to come up with two points to calculate the slope.

You will get your two points by taking the bounds of your interval and substitute those x-values into your equation to find the y-values. Then use the slope formula to calculate the slope.

Remember slope is: $\frac{rise}{run}$ or $\frac{y_2 - y_1}{x_2 - x_1}$

Practice: Calculate the average rate of change of the function $y = (x - 4)^2$ on the given intervals:

$1 \le x \le 3$ $-2 \le x \le 2$

Practice: Calculate the average rate of change of the function $y = x^2 + 4x - 12$ on the given intervals:

$-2 \le x \le 4$ $-3 \le x \le -6$

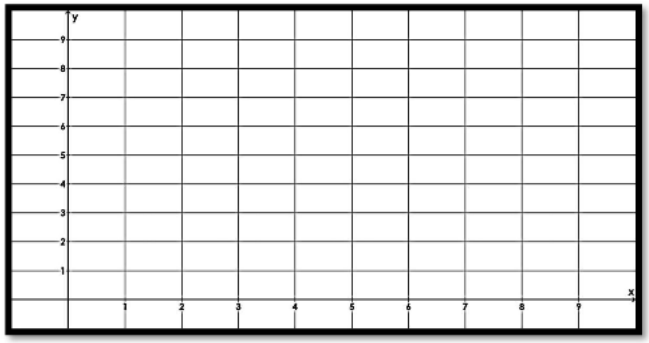
Day 11 – Applications of the Vertex

Words that Indicate Finding Vertex	**Quadratic Equations**
• Minimum/Maximum • Minimize/Maximize • Least/Greatest • Smallest/Largest	Standard Form: $y = ax^2 + bx + c$ y-int: (0, c) Vertex Form: $y = a(x - h)^2 + k$ vertex: (h, k) Factored Form: $y = a(x - p)(x - q)$ x-int: (p, 0) & (q, 0) Vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

1. *The arch of a bridge forms a parabola modeled by the function $y = -0.2(x - 40)^2 + 25$, where x is the horizontal distance (in feet) from the arch's left end and y is the corresponding vertical distance (in feet) from the base of the arch. How tall is the arch?*

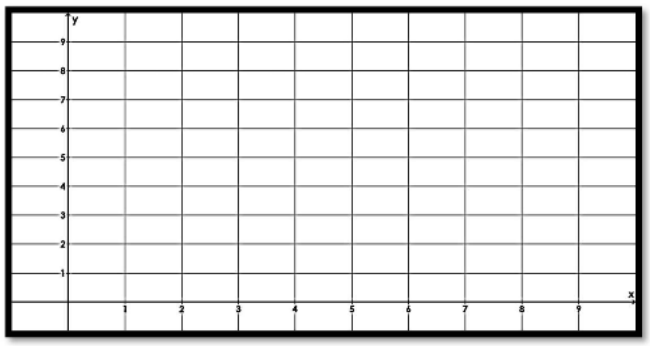
2. *Suppose the flight of a launched bottle rocket can be modeled by the equation $y = -x^2 + 6x$, where y measures the rocket's height above the ground in meters and x represents the rocket's horizontal distance in meters from the launching spot at x = 0.*

a. How far has the bottle rocket traveled horizontally when it reaches it maximum height? What is the maximum height the bottle rocket reaches?



b. How far does the bottle rocket travel in the horizontal direction from launch to landing?

3. *A frog is about to hop from the bank of a creek. The path of the jump can be modeled by the equation* $h(x) = -x^2 + 4x + 1$, *where* $h(x)$ *is the frog's height above the water and* x *is the number of seconds since the frog jumped. A fly is cruising at a height of 5 feet above the water. Is it possible for the frog to catch the fly, given the equation of the frog's jump?*



4. *A baker has modeled the monthly operating costs for making wedding cakes by the function* $y = 0.5x^2 - 12x + 150$, *where* y *is the total costs in dollars and* x *is the number of cakes prepared.*

a. How many cakes should be prepared each month to yield the minimum operating cost?

b. What is the minimum monthly operating cost?

5. A street vendor sells about 20 shirts a day when she charges $8 per shirt. If she decreases the price by $1, she sells about 10 more shirts each day.

a. How many shirts does she have to sell to maximize her revenue? What is her maximum revenue?

Price	Number of Shirts Sold	Revenue
$8	20	

b. How much more will she make a day?

c. Write a quadratic function that models the scenario.

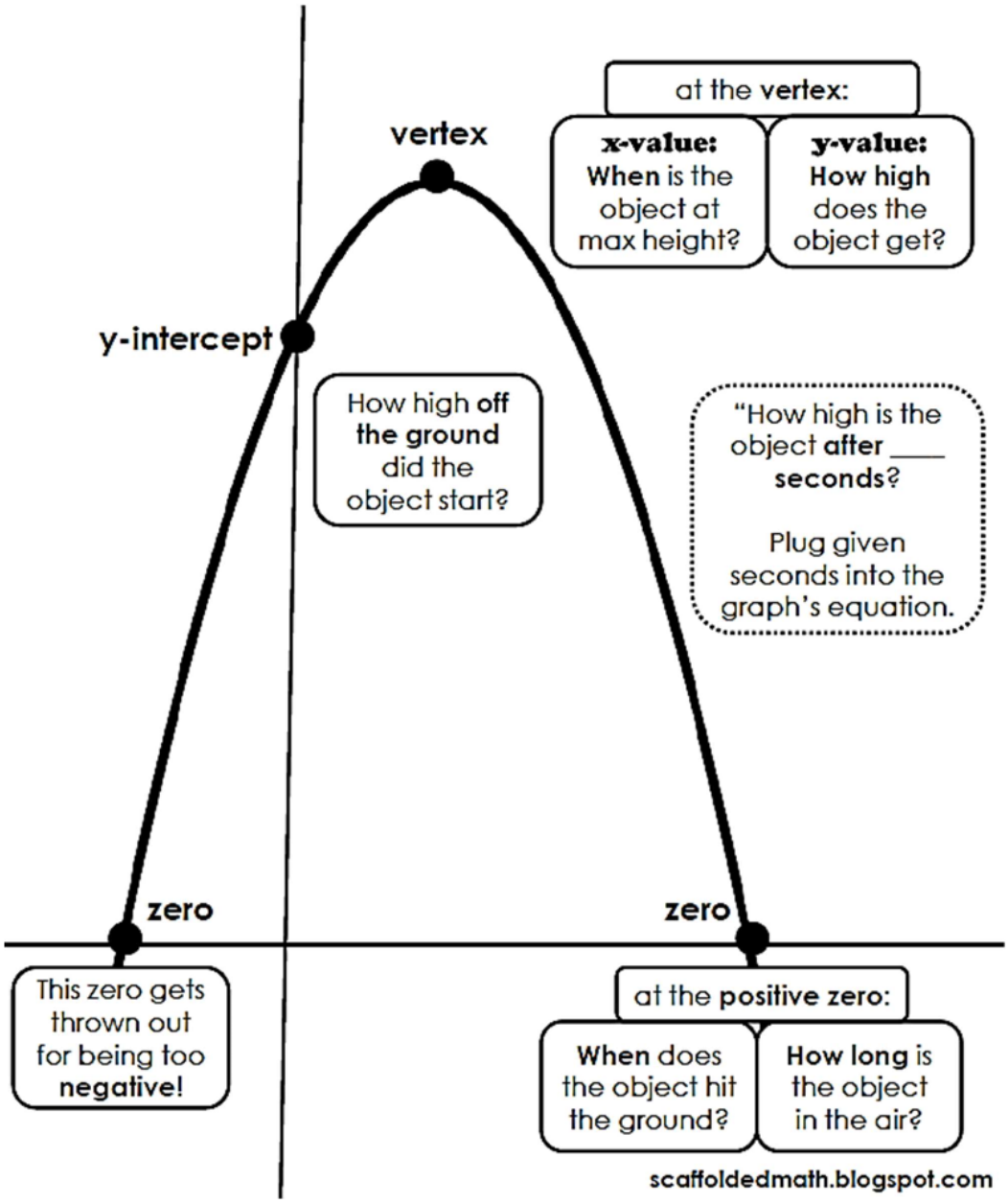
6. You run a canoe rental business on a small river in Georgia. You currently charge $12 per hour canoe and average 36 rentals a day. An industry journal says that for every fifty cent increase in rental price, the average business can expect to lose two rentals a day.

a. Use this information to attempt to maximize your income. What should you charge?

Price	Number of Rentals	Revenue
$12	36	

b. Write a quadratic function that models the scenario.

Quadratic Keywords



Day 12 – Comparing Quadratic Functions

When comparing quadratic functions, you will want to look at their different characteristics (such as the vertices, y-intercepts, zeros, etc). Most of the time, when you are asked to compare different quadratic functions, they will be in different representations (table, graphs, equations, or word problems).

Example 1: Which quadratic function has the bigger y-intercept?

a. $y = x^2 + 4x + 7$

b.

X	-4	-3	-2	-1	0	1
y	0	-1	0	3	8	15

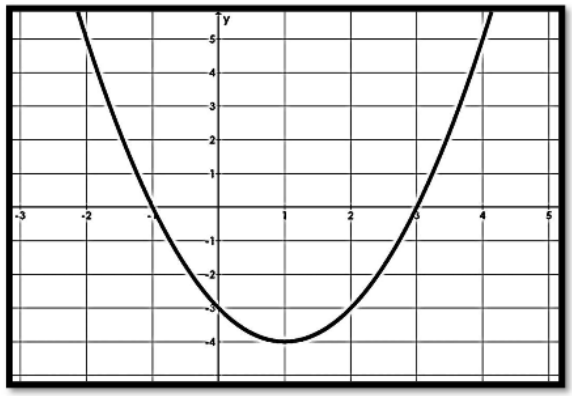
Example 2: Which quadratic functions have an x-intercept at (3, 0)?

a. $y = (x + 3)(x + 1)$

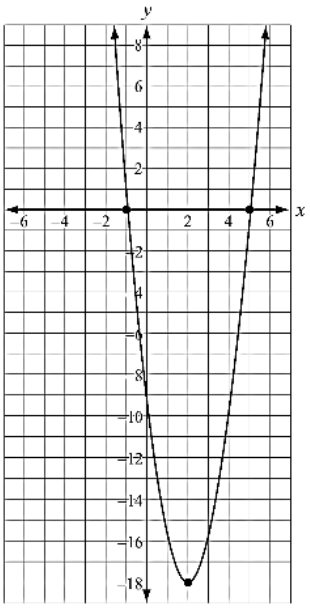
b.

X	-1	0	1	2	3	4
y	8	3	0	-1	0	3

c.



Example 3: This graph shows a function $f(x)$. Compare the graph of $f(x)$ to the graph of the function given by the equation $g(x) = 4x^2 + 6x - 18$. Which function has the lesser minimum value? How do you know?



Example 4: Three students are shooting wads of paper with a rubber band, aiming for a trash can in the front of the room. The height of each student's paper wad, in feet, is given as a function of the time in seconds. Which student's paper wad flies the highest? *(Adopted from Walch Analytic Geometry)*

- The path of Alejandro's paper was is modeled by the equation $f(x) = -x^2 + 2x + 7$
- After 3 seconds, Connor's paper wad achieves a maximum height of 6.5 feet above the floor.
- Melissa's paper wad is estimated to reach the heights shown in the table below.

x	0	2	3	4
y	3	6	7	6

Example 5: For the function $g(x) = (x - 3)^2 - 2$, is the average rate of change greater between $x = 0$ and $x = 1$ OR between $x = 1$ and $x = 2$?