

The Discriminant

Instead of observing a quadratic function's graph and/or solving it by factoring, there is an alternative way to determine the number of real solutions called the **discriminant**.

<p>Given a quadratic function in standard form: $ax^2 + bx + c = 0$, where $a \neq 0$,</p> <p>The discriminant is found by using: $b^2 - 4ac$</p> <p>This value is used to determine the number of real solutions/zeros/roots/x-intercepts that exist for a quadratic equation.</p>	<p>Interpretation of the Discriminant ($b^2 - 4ac$)</p> <ul style="list-style-type: none"> + If $b^2 - 4ac$ is positive: 2 • If $b^2 - 4ac$ is zero: - If $b^2 - 4ac$ is negative: none
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Practice: Find the discriminant for the previous three functions:

a.) $f(x) = x^2 - 4x + 3$

a = _____ b = _____ c = _____

Discriminant: _____

#. of real solutions: _____

b.) $f(x) = x^2 + 10x + 25$

a = _____ b = _____ c = _____

Discriminant: _____

of real zeros: _____

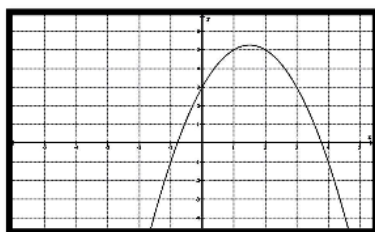
c.) $f(x) = x^2 + x + 1$

a = _____ b = _____ c = _____

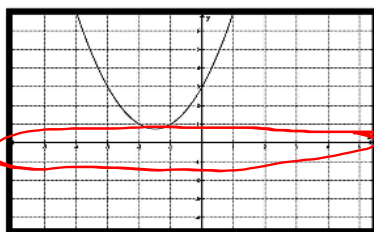
Discriminant: _____

of real roots: _____

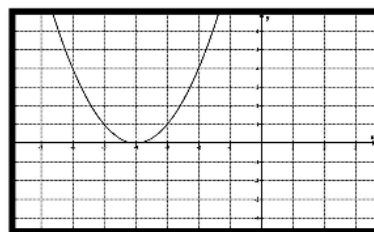
Practice: Determine whether the discriminant would be greater than, less than, or equal to zero.



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Quadratic Conundrum

Consider the quadratic equation $x^2 + 3x - 3 = 0$.

a. How many zeros does this function have?

2

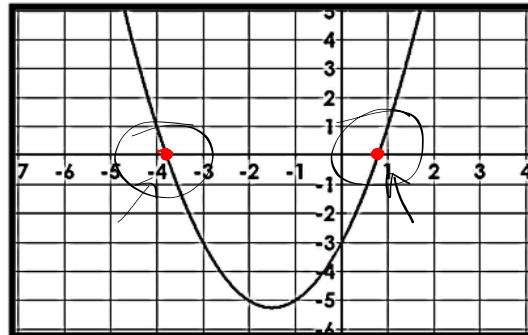
b. Calculate the discriminant: 21

$$(3)^2 - 4(1)(-3) = 21$$

c. If possible, factor the quadratic equation.

~~$x^2 + 3x - 3$~~

~~$(x + 1)(x - 3)$~~ not possible



d. How many solutions does the discriminant of this function imply it would have? Were you able to find these solutions by factoring?

(+) = 2 solutions

The Quadratic Formula

We have learned three methods for solving quadratics: factoring, taking the square root, and completing the square. Factoring quadratics only works when the equations are factorable. Taking the square root only works when the equations are not in standard form. Completing the square only works when a is 1 and b is even.

What method do you use when your equations are not factorable, but are in standard form, and a may not be 1 and b may not be even?

The Quadratic Formula
for equations in standard form: $y = ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

x represents the zeros and $b^2 - 4ac$ is the discriminant

Practice with the Quadratic Formula

For the quadratic equations below, use the quadratic formula to find the solutions. Write your answer in simplest radical form.

$$x = \frac{-b \pm \sqrt{\quad}}{2a}$$

1) $4x^2 - 13x + 3 = 0$ $a = 4$ $b = -13$ $c = 3$

$b^2 - 4ac = (-13)^2 - 4(4)(3)$
 $169 - 48 = 121$

$$x = \frac{-(-13) \pm \sqrt{121}}{2(4)}$$

$$x = \frac{13 \pm 11}{8}$$

Discriminant: 121 \rightarrow 2 solutions
 Solutions: $x = 3, x = \frac{1}{4}$
 Approx: _____

2) $9x^2 + 6x + 1 = 0$ $a = 9$ $b = 6$ $c = 1$

$b^2 - 4ac \Rightarrow (6)^2 - 4(9)(1)$
 disc $\boxed{0}$

$$\frac{-b \pm \sqrt{\text{disc}}}{2a} \Rightarrow \frac{-6 \pm \sqrt{0}}{2(9)}$$

$$\frac{-6}{18} = \boxed{-\frac{1}{3}}$$

Discriminant: 0
 Zeros: $x = -1/3$
 Approx: _____

3) $7x^2 + 8x + 3 = 0$ $a = 7$ $b = 8$ $c = 3$

$b^2 - 4ac = (8)^2 - 4(7)(3)$
 $64 - 84 = -20$

~~$x = \frac{-8 \pm \sqrt{-20}}{2(7)}$~~

Discriminant: -20 \rightarrow no solutions
 x = none
 Approx: _____

4) $-3x^2 + 2x - 8 = 0$ $a = -3$ $b = 2$ $c = 8$

\star $-3x^2 + 2x + 8 = 0$ \star
 $b^2 - 4ac \Rightarrow (2)^2 - 4(-3)(8)$

$$x = \frac{-b \pm \sqrt{\text{disc}}}{2a} = \frac{-2 \pm \sqrt{100}}{2(-3)}$$

Discriminant: 100
 Roots: $\frac{-4}{3} = 2$
 Approx: n/a

$$\frac{-2+10}{-6} = \frac{8}{-6} = -\frac{4}{3}$$

$$\frac{-2-10}{-6} = \frac{-12}{-6} = 2$$

Algebra 1

Unit 9 - Quadratic Equations

Notes

5) $6x^2 + 3 = 10x$
 $\quad \quad \quad \underline{-10x} \quad \underline{-10x}$
 $a = 6 \quad b = -10 \quad c = 3$

6) $\frac{1}{2}x^2 + 6x + 13 = 0$ $a = \underline{\quad} \quad b = \underline{\quad} \quad c = \underline{\quad}$

28
 \wedge
 14 2
 $\circ \circ$
 $\sqrt{2 \cdot 2 \cdot 7}$

$6x^2 - 10x + 3 = 0$

$b^2 - 4ac$
 $(-10)^2 - 4(6)(3)$
 $\boxed{28}$

$\frac{-b \pm \sqrt{\text{disc.}}}{2a} \Rightarrow \frac{10 \pm \sqrt{28}}{2(6)}$

$\frac{5 \cancel{10} \pm \cancel{2}\sqrt{7}}{\cancel{12} 6}$

$\frac{5 \pm \sqrt{7}}{6}$

Discriminant: 28

Solutions: _____

Approx: _____

Discriminant: _____

Zeros: _____

Approx: _____

$\frac{5 + \sqrt{7}}{6} \quad \& \quad \frac{5 - \sqrt{7}}{6}$

ex. $\frac{-8 \pm 3\sqrt{2}}{4}$

ex. $\frac{3 \pm \sqrt{3}}{6}$