Day 4 – Irrational and Rational Numbers

Rational Numbers:

- Can be expressed as the quotient of two integers (i.e. a fraction) with a denominator that is not zero.
- Counting/Natural, Integers, Fractions, and Terminating & Repeating decimals are rational numbers.
- Many people are surprised to know that a repeating decimal is a rational number. $0.\overline{3}$
- $\sqrt{9}$ is rational - you can simplify the square root to 3 which is the quotient of the integers 3 and 1.

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Examples: -5, 0, 7, 3/2, $0.\overline{26}$

Irrational Numbers: CRAZY

- Can't be expressed as the quotient of two integers (i.e. a fraction) such that the denominator is not zero.
- If your number contains π, a radical (not a perfect square), or a decimal that goes on forever (does not repeat), it is an irrational number.

Examples: $\sqrt{7}$, $\sqrt{5}$, π, 4.569284….

Practice: Classify each number as rational or irrational and explain why.

a. $\sqrt{15}$

15
5 3

I

b. ¼

R

c. $\sqrt{2}\cdot\sqrt{18}$

$\sqrt{36}$

6 R

d. $\sqrt{25}+\sqrt{1}$

$5+1=6$

R

e. $\sqrt{7}+\sqrt{28}$

28
4 7
2 2

$\sqrt{7}+\sqrt{28}$

$\sqrt{7}+\sqrt{2\cdot 2\cdot 7}$

$1\sqrt{7}+2\sqrt{7}$

$\boxed{3\sqrt{7}}$

I

f. $\pi+(-\pi)$

0

R

9

Adding Rational and Irrational Numbers

		Rational		
	+	5	½	0
Rational	5	10	5.5	5
	½	5.5	1	½
	0	5	½	0

Adding Two Rational Numbers

Conclusion:

The sum of two rational numbers is always rational.

		Rational		
	+	5	½	0
Irrational	$\sqrt{2}$	$5+\sqrt{2}$	$\frac{1}{2}+\sqrt{2}$	$\sqrt{2}$
	$-\sqrt{2}$	$5-\sqrt{2}$	$\frac{1}{2}-\sqrt{2}$	$-\sqrt{2}$
	π	$5+\pi$	$\frac{1}{2}+\pi$	π

Adding Rational and Irrational Numbers

Conclusion:

The sum of a rational and irrational is always irrational

		Irrational		
	+	$\sqrt{2}$	$-\sqrt{2}$	π
Irrational	$\sqrt{2}$	$\sqrt{2}+\sqrt{2}$ $2\sqrt{2}$	0	$\pi+\sqrt{2}$
	$-\sqrt{2}$	0	$-\sqrt{2}-\sqrt{2}$ $-2\sqrt{2}$	$\pi-\sqrt{2}$
	π	$\sqrt{2}+\pi$	$-\sqrt{2}+\pi$	2π

Adding Two Irrational Numbers

Conclusion:

The sum of two irrational numbers is mostly irrational

Except when: when = 0 (opposite)

Multiplying Rational and Irrational Numbers

	Rational		
x	5	½	-1
5	25	5/2	-5
½	5/2	1/4	-1/2
-1	-5	-1/2	1

Row label: Rational ~~Irrational~~

Multiplying Two Rational Numbers

Conclusion:

The product of two rational numbers is always rational

	Rational		
x	5	½	-1
$\sqrt{2}$	$5\sqrt{2}$	$\frac{\sqrt{2}}{2}$	$-\sqrt{2}$
$-\sqrt{2}$	$-5\sqrt{2}$	$-\frac{\sqrt{2}}{2}$	$\sqrt{2}$
π	5π	$\frac{\pi}{2}$	$-\pi$

Row label: Irrational

Multiplying Rational and Irrational Numbers

Conclusion:

The product of a rational and irrational is always irrational

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2 2

$\sqrt{2 \cdot 2} = 2$

	Irrational		
x	$\sqrt{2}$	$-\sqrt{2}$	π
$\sqrt{2}$	$\sqrt{4}$ 2	-2	$\pi\sqrt{2}$
$-\sqrt{2}$	-2	2	$-\pi\sqrt{2}$
π	$\pi\sqrt{2}$	$-\pi\sqrt{2}$	π^2

Row label: Irrational

Multiplying Two Irrational Numbers

Conclusion:

The product of two irrational numbers is sometimes irrational

Except when: multiply same √#

*If you ever multiply an irrational number by 0 (which is a rational number), your outcome will always be 0, which is a rational number. Most of the time, when multiplying, it will say a nonzero rational number, which means 0 is excluded from the rational number set.

Ex. $\sqrt{2} \cdot 0 = 0$ Ex. $\pi \cdot 0 = 0$