Exponential Functions

Day 1 – Graphing Exponential Functions

Exploring with Graphs: Graph the following equations:

a. $y = 2x$

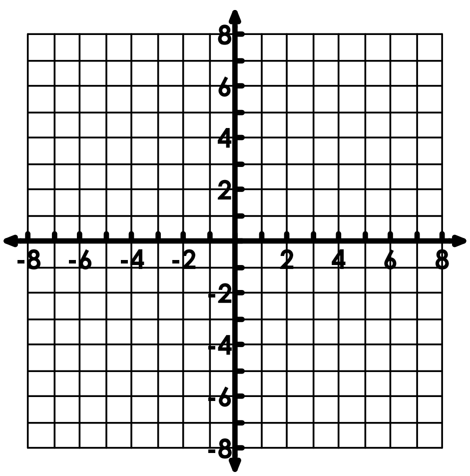
x	-3	-2	-1	0	1	2	3
y							

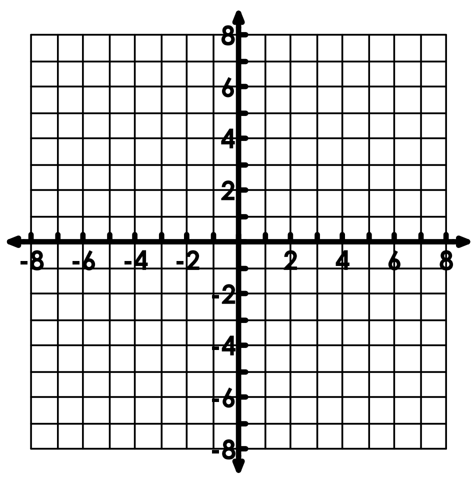
b. $y = x^2$

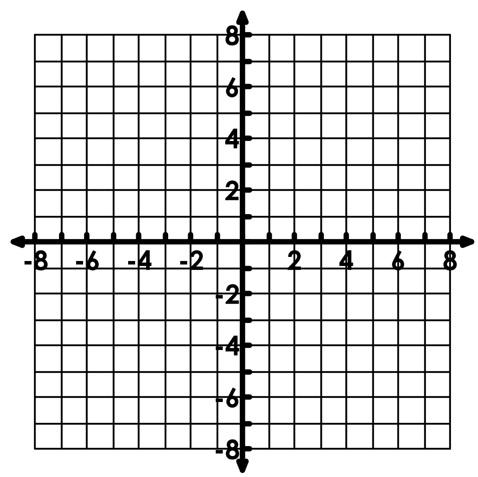
x	-3	-2	-1	0	1	2	3
y							

c. $y = 2^x$

x	-3	-2	-1	0	1	2	3
y							







Type: ______________________ Type: ______________________ Type: ______________________

How is Equation C different from Equations A and B (you have already learned about equations A & B).

Graphing Exponential Functions

The general form of an exponential function is:

$$y = ab^x$$

Where **a** represents your starting or initial value/population and y-intercept
b represents your growth/decay factor

When you graph exponential functions, you will perform the following steps:

Graphing Exponential Functions Steps

1. Create an x-y chart with 5 values for x (Use table feature to pick 5 values)
2. Substitute those values into the function and record the y or f(x) values.
3. Graph each ordered pair on a graph.

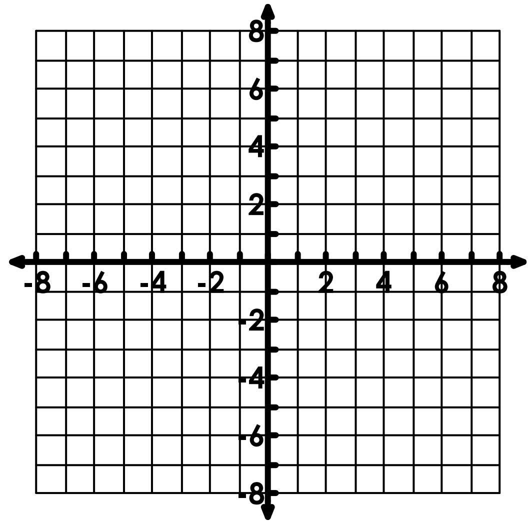
Graph the following:

a. $y = 3(4)^x$

x	y

Y-intercept:

Asymptote:

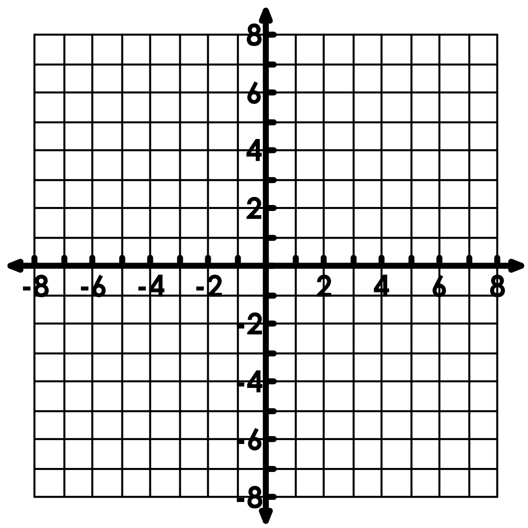


b. $f(x) = 2^x$

x	y

Y-intercept:

Asymptote:

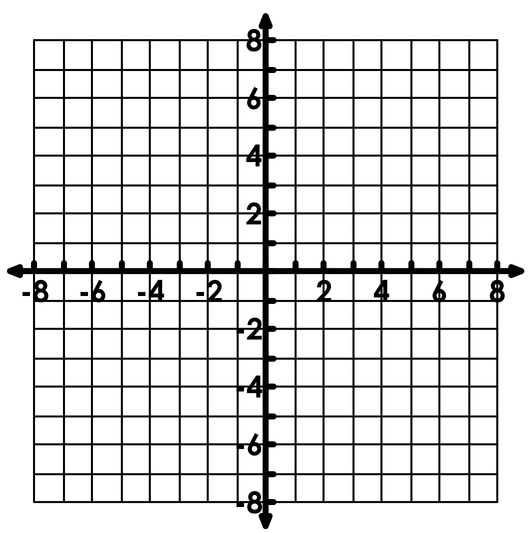


c. $y = 3\left(\frac{1}{2}\right)^x$

x	y

Y-intercept:

Asymptote:

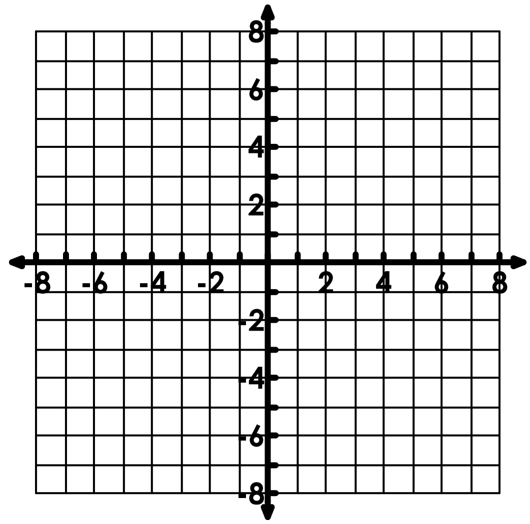


d. $f(x) = 4(.25)^x$

x	y

Y-intercept:

Asymptote:



Think about it…

You have two ways you can find the y-intercept when given an equation: $y = 3(4)^x$

1.

2.

Summary of Different Types of Exponential Graphs

Equation	'a' values	'b' values	General Shape of Graph
$y = 3(4)^x$ $f(x) = 2^x$			
$y = 3\left(\frac{1}{2}\right)^x$ $f(x) = 4(.25)^x$			

Day 2 – Transformations of Exponential Functions

Transformations of exponential functions is very similar to transformations with quadratic functions. Do you remember what a, h, and k do to the quadratic function?

A: ______________________ H: ______________________ K: ______________________

Summary of Exponential Transformations

The general form of an exponential function is:

$$f(x) = a(b)^{x-h} + k.$$

***When your graph is shifted vertically, the y-intercept becomes a + k.**
***When the graph is shifted vertically, the asymptote becomes y = k.**

If h is **positive**, the graph…

In the equation, I would see…

If h is **negative**, the graph…

In the equation, I would see…

If **a** is **negative**, the graph…

$$y = a(b)^{x-h} + k$$

If **a** is **between 0 and 1**, the graph…

Grows ____________

If **a** is **greater than 1**, the graph…

Grows ____________

If **b** is **greater than 1…**

If **b** is **between 0 & 1…**

If k is **positive**, the graph…

If k is **negative**, the graph…

Asymptote:

Practice Identifying Transformations

Example: Describe the transformations from the parent function to the transformed function:

A. $f(x) = 3^x \rightarrow f(x) = 3^{x+3}$

B. $y = (5)^x \rightarrow y = \frac{1}{2}(5)^x - 4$

C. $y = (0.4)^x \rightarrow y = -3(0.4)^x + 8$

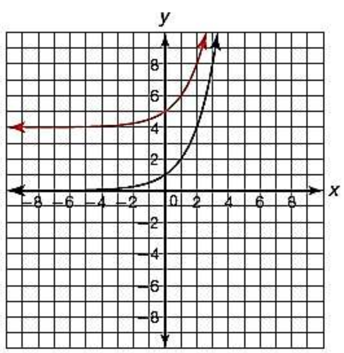
D. $f(x) = 3^x \rightarrow f(x) = \frac{3}{4}(3)^{x-2}$

E. $y = 5^x \rightarrow y = -\frac{1}{2}(5)^{x+2}$

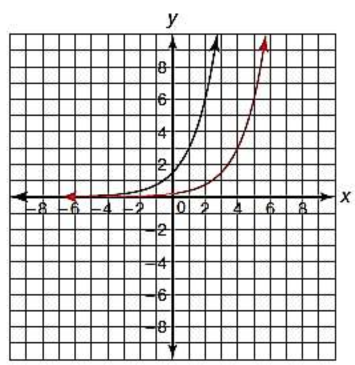
F. $y = 0.4^x \rightarrow y = 2(0.4)^x - 6$

Example: Using the graphs of f(x) and g(x), described the transformations from f(x) to g(x):

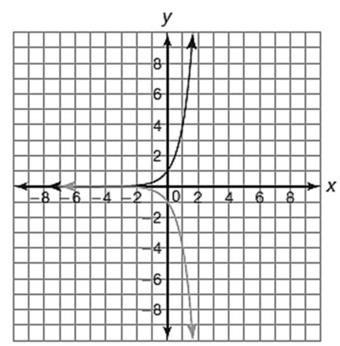
A.



B.



C.



Example: Using the function $g(x) = 5^x$, create a new function h(x) given the following transformations:

A. up 4 units

B. left 2 units

C. down 7 units and right 3 units

D. stretch by 3

E. reflect over x-axis and left 3

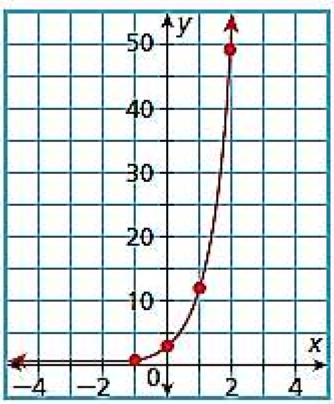
F. Shrink by ½ and reflect over x-axis

Day 3 – Characteristics of Exponential Functions

As you can hopefully recall, you learned about characteristics of functions in Unit 2 with linear functions and Unit 5 with quadratic functions. We are going to apply the same characteristics, but this time to exponential functions.

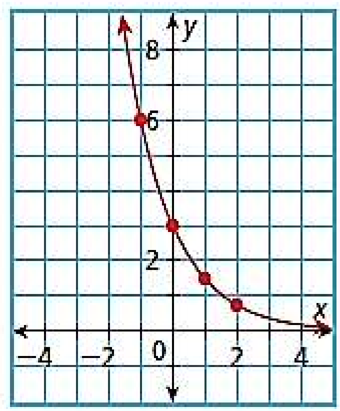
Domain and Range

Domain		
Define: All possible values of x	**Think:** How far left to right does the graph go?	**Write:** Smallest x ≤ x ≤ Biggest x *use < if the circles are open*
Range		
Define: All possible values of y	**Think:** How far down to how far up does the graph go?	**Write:** y < highest y value (opens down) y > lowest y value (opens up)



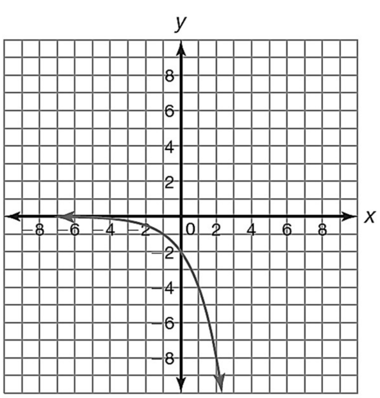
Domain:

Range:



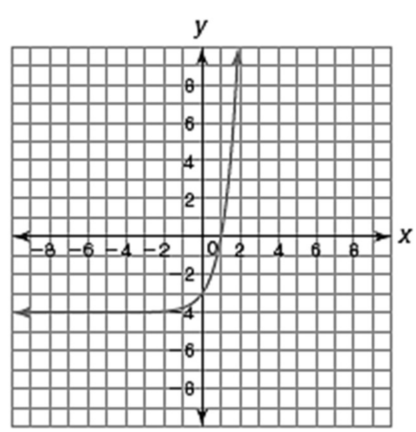
Domain:

Range:



Domain:

Range:

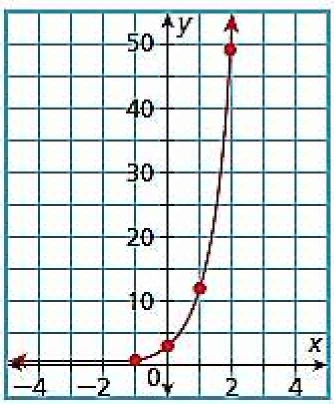


Domain:

Range:

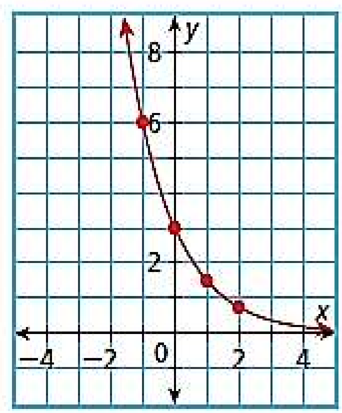
Intercepts and Zeros

Y-Intercept		
Define: Point where the graph crosses the y-axis	**Think:** At what coordinate point does the graph cross the y-axis?	**Write:** (0, b)
X-Intercept		
Define: Point where the graph crosses the x-axis	**Think:** At what coordinate point does the graph cross the x-axis?	**Write:** (a, 0)
Zero		
Define: Where the function (y-value) equals 0	**Think:** At what x-value does the graph cross the x-axis?	**Write:** x = ____



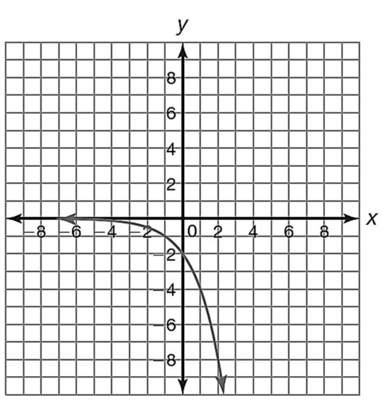
X-intercept: Zero:

Y-intercept:



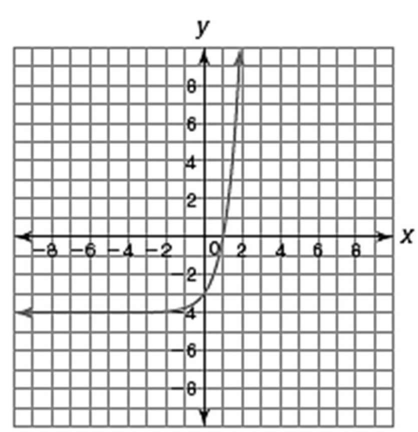
X-intercept: Zero:

Y-intercept:



X-intercept: Zero:

Y-intercept:

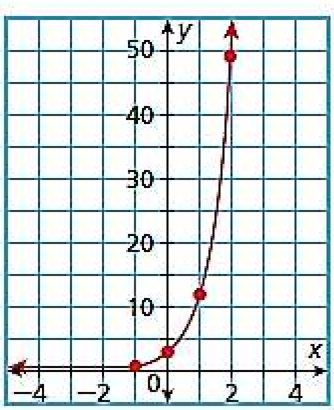


X-intercept: Zero:

Y-intercept:

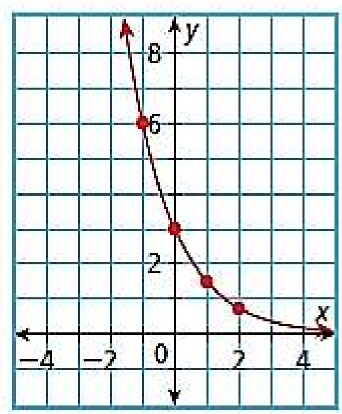
Extremas and Asymptotes

<table>
<tr><th colspan="3">Maximum</th></tr>
<tr><td>Define:
Highest point of a function.</td><td>Think:
What is my highest point on my graph?</td><td>Write:
y =</td></tr>
<tr><th colspan="3">Minimum</th></tr>
<tr><td>Define:
Lowest point of a function.</td><td>Think:
What is the lowest point on my graph?</td><td>Write:
y =</td></tr>
<tr><th colspan="3">Asymptotes</th></tr>
<tr><td>Define:
A line that the graph get closer and closer to, but never touches or crosses.</td><td>Think:
What values does my graph begin to flat line towards?</td><td>Write:
y =</td></tr>
</table>



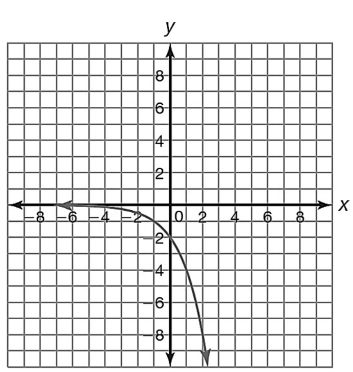
Maximum: Minimum:

Asymptote:



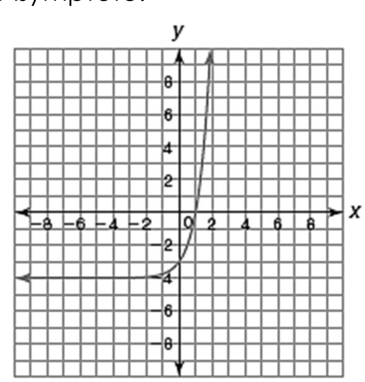
Maximum: Minimum:

Asymptote:



Maximum: Minimum:

Asymptote:



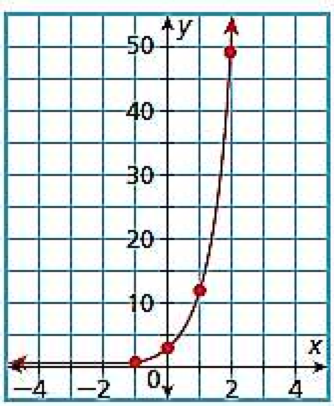
Maximum: Minimum:

Asymptote:

Intervals of Increase and Decrease

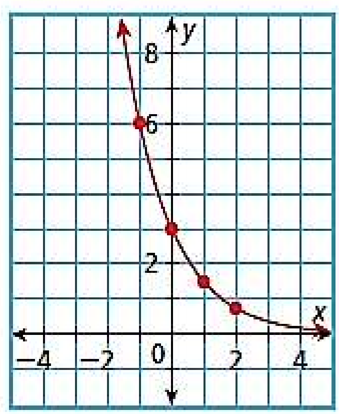
Interval of Increase		
Define: The part of the graph that is rising as you read left to right.	**Think:** From left to right, is my graph going up?	**Write:** An inequality using the x-value of the vertex

Interval of Decrease		
Define: The part of the graph that is falling as you read from left to right.	**Think:** From left to right, is my graph going down?	**Write:** An inequality using the x-value of the vertex



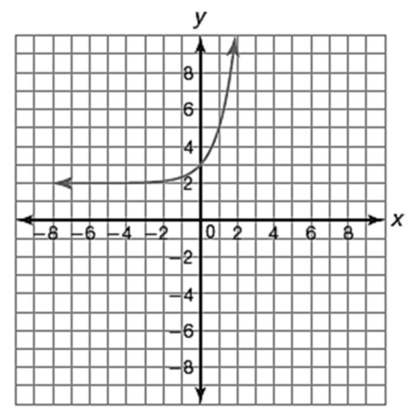
Interval of Increase:

Interval of Decrease:



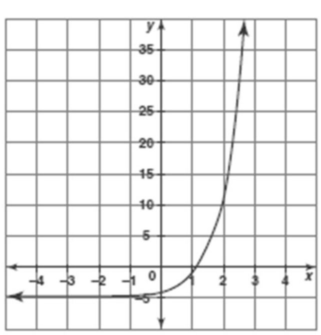
Interval of Increase:

Interval of Decrease:



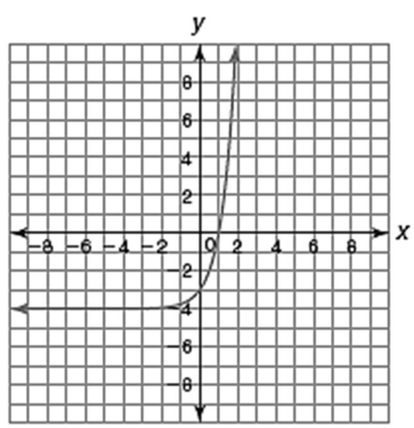
Interval of Increase:

Interval of Decrease:



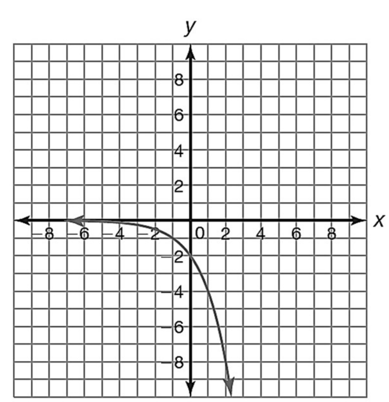
Interval of Increase:

Interval of Decrease:



Interval of Increase:

Interval of Decrease:



Interval of Increase:

Interval of Decrease:

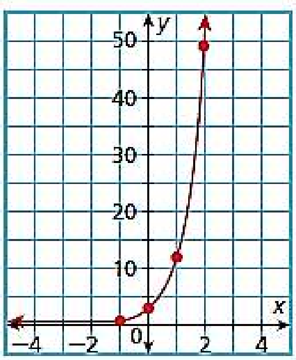
End Behavior

End Behavior

Define:

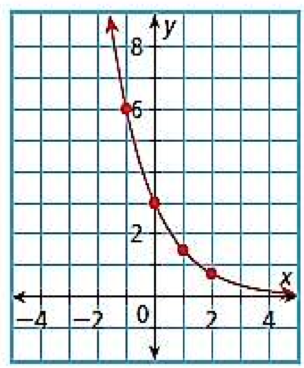
Behavior of the ends of the function (what happens to the y-values or f(x)) as x approaches positive or negative infinity. The arrows indicate the function goes on forever so we want to know where those ends go.

Think: As x goes to the left (negative infinity), what direction does the left arrow go?	**Write:** As x → -∞, f(x) → _____
Think: As x goes to the right (positive infinity), what direction does the right arrow go?	**Write:** As x → ∞, f(x) → _____



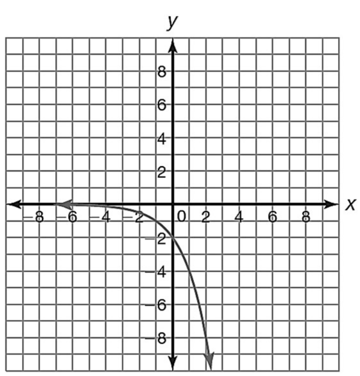
As x approaches -∞, f(x) approaches _____.

As x approaches ∞, f(x) approaches _____.



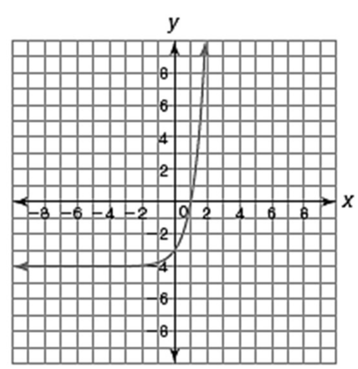
As x approaches -∞, f(x) approaches _____.

As x approaches ∞, f(x) approaches _____.



As x approaches -∞, f(x) approaches _____.

As x approaches ∞, f(x) approaches _____.

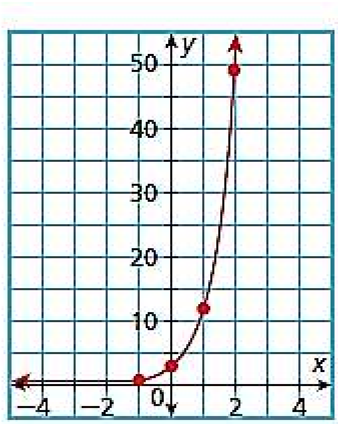


As x approaches -∞, f(x) approaches _____.

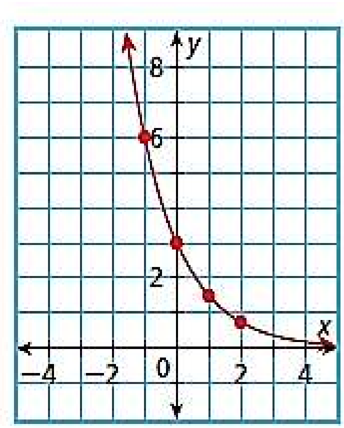
As x approaches ∞, f(x) approaches _____.

Average Rate of Change from a Graph

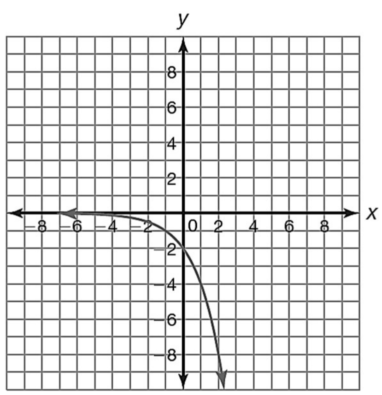
Average Rate of Change: Rate of change or slope for a given interval on a graph. The given interval is written using the inequality notation $a \le x \le b$, where a and b represent the initial and final x-value of the interval.



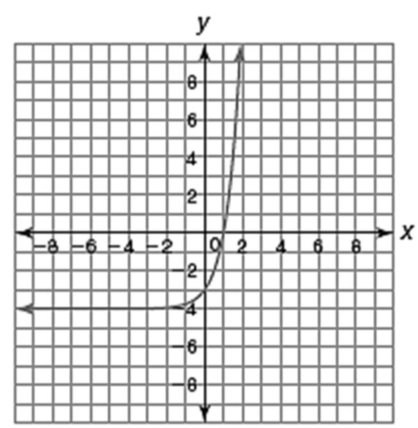
Calculate the average rate of change for the interval $0 \le x \le 2$



Calculate the average rate of change for the interval $-1 \le x \le 2$



Calculate the average rate of change for the interval $0 \le x \le 2$



Calculate the average rate of change for the interval $0 \le x \le 1$

Average Rate of Change from an Equation

If you are given an equation of a function and asked to calculate the average rate of change for that function over a given interval, you will substitute the initial x-value and the final x-value into the function to create two sets of ordered pairs. Then using the ordered pairs, substitute into the slope formula.

a. $y = 3^x$; $1 \le x \le 3$

b. $y = 2(1/2)^x$; $-4 \le x \le 0$