Unit 4: Equations & Inequalities

Learning Goal 4.1 – Solving Equations

After completion of this unit, you will be able to…

Learning Target #1: Creating and Solving Linear Equations

- Solve one, two, and multi-step equations (variables on both sides)
- Justify the steps for solving a linear equation
- Create and solve an equation from a context
- Solve a literal equation (multiple variables) for a specified variable
- Use a Formula to Solve Problems

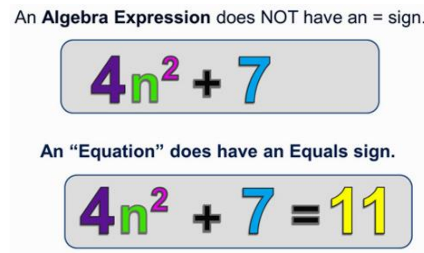
Monday	Tuesday	Wednesday	Thursday	Friday
		2nd Day 1 Solving 1 & 2 Step EquationS	**3rd** Day 2 Multi-Step Equations	**4th** Day 3 Multi-Step Equations, Properties of Equality
7th Day 4 Solving Literal Equations	**8th** Day 5 Solving Literal Equations	**9th** Day 6 Creating & Solving Equations from a Context	**10th** EARLY RELEASE (1st and 2nd) Practice Day	**11th** Day 7 Creating Equations from a Context
14th Day 9 **4.1 Assessment,** Day 8 - Graphing Inequalities	**15th** Day 9 Solving 1 and 2 Step Inequalities	**16th** PSAT DAY (3rd and 4th) Practice Day	**17th** Day 10 Solving MultiStep Inequalities	**18th** Day 11 Creating Inequalities from a Context
21st Day 12 Creating Inequalities from a Context	**22nd** **4.2 Assessment**	**23rd**	**24th**	**25th**

	Monday	Tuesday	Wednesday	Thursday	Friday
AM	**NONE**	**Mrs. Jackson 7:45 – 8:15 Room 1210**	**Mr. Webb 7:45 – 8:15 Room 1205**	**Mr. Watson 7:45 – 8:15 Room 1208**	**Mr. Watson 7:45 – 8:15 Room 1208**
PM	**Mrs. Petersen 3:30 – 4:30 Room 1210**	**Mr. Webb 3:30 – 4:30 Room 1205**	**NONE**	**Mrs. Jackson 3:30 – 4:30 Room 1210**	**NONE**

Day 1 – Solving One & Two Step Equations

Remember, an **expression** is a mathematical "phrase" composed of terms, coefficients, and variables that stands for a single number, such as 3x + 1 or $x^2 - 1$. We use Properties of Operations to *simplify* algebraic expressions. Expressions do NOT contain equal signs.

An **equation** is a mathematical "sentence" that says two expressions are equal to each other such as 3x + 1 = 5. We use Properties of Equality (inverse operations) to *solve* algebraic equations. Equations contain equal signs.



When solving equations, you must perform **inverse operations**, which means you have to perform the operation opposite of what you see. You must also remember the operation you perform on one side of the equation must be performed to the other side.

Informal		Formal		
Operation	**Inverse**	**Property**	**General Example**	**Example 1**
Addition		Addition Property of Equality	If a = b, then a + c = b + c	If x – 4 = 8, then x = 12
Subtraction		Subtraction Property of Equality	If a = b, then a – c = b - c	If x + 5 = 7, then x = 2
Multiplication		Multiplication Property of Equality	If a = b, then ac = bc	If $\frac{x}{2} = 9$, then x = 18
Division		Division Property of Equality	If a = b, then $\frac{a}{c} = \frac{b}{c}$	If 2x = 10, then x = 5

No More "Cancelling"

When you first learned to solve equations in middle school, you might have used the words "cancel". We are no longer going to use the word "cancel". Take a look at the following examples:

$x - 120 = 80$
$+120 \quad +120$ ← **Adding the opposite**
$x = 200$

Additive inverse
Adding to zero

$\frac{k}{2} = 16$
$\frac{k}{\cancel{2}} \times \cancel{2} = 16 \times 2$ ← **Multiplying by the Reciprocal**
$k = 32$ ✓

Multiplicative Inverse
Divides/Multiplies to one

Additive Inverse	A number plus its inverse equals 0.	a + -a = 0	7 + -7 = 0
Multiplicative Inverse (Reciprocal)	A number times its reciprocal equals 1.	$a \cdot \frac{1}{a} = 1$	$3 \cdot \frac{1}{3} = 1$

Solving One Step Equations Practice

Practice: Solve each equation.

1. $x - 4 = 3$ Operation You See: _______________ Inverse Operation: _______________

2. $y + 4 = 3$ Operation You See: _______________ Inverse Operation: _______________

3. $\frac{s}{3} = 9$ Operation You See: _______________ Inverse Operation: _______________

4. $6p = 12$ Operation You See: _______________ Inverse Operation: _______________

Practice: Solve each equation on your own.

a. $x - 6 = 10$

b. $-5d = 25$

c. $8 + m = -4$

d. $\frac{x}{7} = 1$

e. $y - (-9) = 2$

f. $\frac{1}{3}x = 6$

Solving Two Step Equations

When solving equations with more than one step, you still want to think about how you can "undo" the operations you see. For the following equations, describe the operations being performed on each variable (go in PEMDAS order). Then describe the inverses using a table.

a. $3x + 5 = 14$

b. $2n - 6 = 4$

c. $\frac{x-2}{4} = 1$

Practice: Solve each equation, showing all steps, for each variable.

1. $3x - 4 = 14$
2. $2x + 4 = 10$
3. $7 - 3y = 22$
4. $0.5m - 1 = 8$
5. $-6 + \frac{x}{4} = -5$
6. $\frac{x-8}{4} = -5$

Error Analysis with Solving Equations

1. William solved the following equation on his homework last night. However, he solved it incorrectly. Describe the mistake William made and what he should have done instead. Then re-solve the equation to find the correct answer.

Mistake: ____

____

____

$$4 = \frac{y}{8} + 1$$
$$32 = y + 1$$
$$31 = y$$

Corrected Solution:

2. Tyler solved the following equation on his homework last night. However, he solved it incorrectly. Describe the mistake Tyler made and what he should have done instead. Then re-solve the equation to find the correct answer.

Mistake: ____

____

____

$$28y + 7 = 21$$
$$28y = 28$$
$$y = 1$$

Corrected Solution:

Day 2 – Solving Multi-Step Equations

Multi-step equations mean you might have to add, subtract, multiply, or divide all in one problem to isolate the variable. When solving multi-step equations, you are using inverse operations, which is like doing PEMDAS in reverse order.

Multi - Step Equations with Combining Like Terms

Practice: Solve each equation, showing all steps, for each variable.

a. $-5n + 6n + 15 – 3n = -3$

b. $3x + 12x – 20 = 25$

c. $-2x + 4x – 12 = 40$

Multi - Step Equations with the Distributive Property

Practice: Solve each equation, showing all steps, for each variable.

a. $2(n + 5) = -2$

b. $4(2x – 7) + 5 = -39$

c. $6x – (3x + 8) = 16$

Multi – Step Equations with Variables on Both Sides

Practice: Solve each equation, showing all steps, for each variable

a. $5p - 14 = 8p + 4$

b. $8x - 1 = 23 - 4x$

c. $5x + 34 = -2(1 - 7x)$

Error Analysis with Solving Equations

1. Rachel solved the following equation on her homework. However, she solved it incorrectly. Describe the mistake Rachel made and what she should have done instead. Then resolve the equation to find the correct answer.

Mistake: ____

$$-2(7 - y) + 4 = -4$$
$$-14 - 2y + 4 = -4$$
$$-10 - 2y = -4$$
$$-2y = 6$$
$$y = -3$$

Correction Solution:

2. Mikayla solved the following equation on her homework. However, she solved it incorrectly. Describe the mistake Mikayla made and what she should have done instead. Then resolve the equation to find the correct answer.

$$2(x + 3) = -3(-x + 1)$$
$$2x + 6 = 3x - 3$$
$$5x + 6 = -3$$
$$5x = -9$$
$$x = -\frac{5}{9}$$

Mistake: ____

Correction Solution:

Day 3 –Justifying the Solving of Equations

Properties of Addition Operations			
Property	**What It Means**	**General Example**	**Example 1**
Commutative Property of Addition	Rearrange the order and the sum will stay the same.	$a + b = b + a$	$2 + 4 = 4 + 2$
Associative Property of Addition	Change the order of the grouping and the sum will stay the same.	$(a + b) + c = a + (b + c)$	$(4 + 6) + 1 = 4 + (6 + 1)$
Additive Identity	Zero added to any number will equal that number.	$a + 0 = a$	$4 + 0 = 4$
Additive Inverse	A number plus its inverse equals 0.	$a + -a = 0$	$7 + -7 = 0$
Properties of Multiplication Operations			
Commutative Property of Multiplication	Rearrange the order and the product will stay the same.	$a \cdot b = b \cdot a$	$5 \cdot 2 = 2 \cdot 5$
Associative Property of Multiplication	Change the order of the grouping and the product will stay the same.	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(3 \cdot 4) \cdot 2 = 3 \cdot (4 \cdot 2)$
Multiplicative Identity	One times any number equals that number.	$a \cdot 1 = a$	$8 \cdot 1 = 8$
Multiplicative Inverse (Reciprocal)	A number times its reciprocal equals 1.	$a \cdot \frac{1}{a} = 1$	$3 \cdot \frac{1}{3} = 1$
Zero Property of Multiplication	Any number times 0 will always equal 0.	$a \cdot 0 = 0$	$7 \cdot 0 = 0$
Distributive Property	Multiply a number to every term within a quantity (parenthesis).	$a(b + c) = ab + ac$	$4(x + 5) = 4x + 4(5)$ $= 4x + 20$

Practice: Each of the following expressions has been simplified one step at a time. Next to each step, identify the property or simplification used in the step.

1. $4 + 5(x + 7)$ **Given**
   $4 + (5x + 35)$ ____________________
   $5x + 4 + 35$ ____________________
   $5x + (4 + 35)$ ____________________
   $5x + 39$ ____________________

2. $4(10x + 2) – 40x$ **Given**
   $40x + 8 – 40x$ ____________________
   $8 + 40x – 40x$ ____________________
   $8 + 0$ ____________________
   8 ____________________

Justifying the Solving of Equations

Properties of Equality		
Property	**General Example**	**Example 1**
Addition Property	If a = b, then a + c = b + c	If x – 4 = 8, then x = 12
Subtraction Property	If a = b, then a – c = b - c	If x + 5 = 7, then x = 2
Multiplication Property	If a = b, then ac = bc	If $\frac{x}{2} = 9$, then x = 18
Division Property	If a = b, then $\frac{a}{c} = \frac{b}{c}$	If 2x = 10, then x = 5
Reflexive Property	a = a	5 = 5
Symmetric Property	If a = b, then b = a	If 2 = x, then x = 2
Transitive Property	If a = b and b = c, then a = c	If x + 2 = y and y = 4x + 3, then x + 2 = 4x + 3
Substitution Property	If x = y, then y can be substituted for x in any expression	If x = 3 and the expression is 2x – 7, then 2(3) - 7

Practice: Using properties of operations and equality, list each property next to each step in the equation solving process.

Example 1

x + 4 = 9	**Given**
x = 5	

Example 2

7 = x - 5	**Given**
12 = x	
x = 12	

Example 3

$\frac{x}{3} = 5$	**Given**
x = 15	

Example 4

6x = 24	**Given**
x = 4	

Justifying the Solutions to Two & Multi-Step Equations

Practice: Identify the property or simplification that is used in each step to solve the equation.

Example 1

3x + 5 = -13	**Given**
3x = -18	
x = -6	

Example 2

12 = 2(x – 4)	**Given**
12 = 2x – 8	
20 = 2x	
10 = x	
x = 10	

Example 3

5n – 3 = 2(n + 3) + 9	**Given**
5n – 3 = 2n + 6 + 9	
5n – 3 = 2n + 15	
3n – 3 = 15	
3n = 18	
n = 6	

Special Types of Solutions

Solve the following equations. What do you notice about the solutions?

a. $2x - 7 + 3x = 4x + 2$

b. $3(x - 5) + 11 = x + 2(x + 5)$

c. $3x + 7 = 5x + 2(3 - x) + 1$

We are going to use the graphing calculators to view what these equations look like. Draw a sketch of the graphs:

Problem A:

Problem B:

Problem C:

Conclusions:

Day 4 – Isolating a Variable

Isolating a variable simply means to solve for that variable or get the variable "by itself" on one side of the equal sign (usually on the left). Sometimes we may have more than one variable in our equations; these type of equations are called **literal equations**. We solve literal equations the same way we solve "regular" equations.

Steps for Isolating Variables

1. Locate the variable you are trying to isolate.
2. Follow the rules for solving equations to get that variable by itself.

Solving an Equation You're Familiar with	Solving a Literal Equation
2x = 10	gh = m solve for h
2x + 5 = 11	ax + b = c solve for x

Practice:

1. Solve the equation for b: a = bh

2. Solve the equation for b: y = mx + b

3. Solve the equation for x: 2x + 4y = 10

4. Solve the equation for m: y = mx + b

5. Solve the equation for w: p = 2l + 2w

6. Solve the equation for a: $\frac{a}{2} - 1 = b$

Your Turn:

7. Solve the equation for y: $6x - 3y = 15$

8. Solve the equation for h: $V = \frac{1}{3}Bh$

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1. You are visiting a foreign county over the weekend. The forecast is predicted to be 30 degrees Celsius. Are you going to pack warm or cold clothes? Use Celsius = $\frac{5}{9}(F - 32)$.

2. The area of a triangle is given by the formula A = ½bh or $A = \frac{bh}{2}$, where b is the base and h is the height.

a. Use the formula given to find the height of the triangle that has a base of 5 cm and an area of 50 cm.

b. Solve the formula for the height.

c. Use the formula from part b to find the height of a triangle that has a base of 5 cm and an area of 50 cm.

Day 5 – Isolating a Variable (Complex)

One of the most important skills you will encounter for the next two units is the ability to take an equation in standard form (Ax + By = C) and solve for y. You had a few problems from yesterday like this, but take some time to practice a few more.

a. $5x - 2y = 8$

b. $-3x + 3y = 6$

c. $-7x - 4y = 12$

Complex Literal Equations

a. $\frac{5x+y}{a} = 2$ for a

b. $c = \frac{3}{4}y + b$ for b

c. $P = \frac{1.2W}{H^2}$ for W

d. $p(t + 1) = -2$, for t

e. $\frac{3ax-n}{5} - 4$ for x

f. $\frac{34-A}{2} = H$ for A

Day 6 – Creating Equations from a Context

Explore: Read the scenario below and answer the following questions.

Annie is throwing a graduation party. She wants to send nice invitations to all of her guests. She found a company that will send her a pack of 10 personalized invitations for $6 each, plus a $5 shipping fee for the entire order, no matter how many invitations she orders.

a. What is the total cost of Annie purchasing three packs of invitations?

b. What is the total cost of Annie purchasing five packs of invitations?

c. What is the total cost of Annie purchasing ten packs of invitations?

d. Describe how you calculated the cost of each order.

e. Write an algebraic expression that represents the total cost of any order. Let p represent the number of invitation packs that were ordered.

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f. How many packs of invitations were ordered if the total cost of the order was $53?

g. How many packs of invitation were ordered if the total cost of the order was $29?

h. Describe how you calculated the number of invitation packs ordered for any order amount.

i. Write an equation to describe this situation. Let p represent the number of invitation packs ordered and c represent the total cost of the order.

j. Use your equation to determine how many invitation packs Annie ordered if her total cost was $47.

Earlier in our unit, you learned to write expressions involving mathematical operations. You used the following table to help you decode those written expressions. We are going to use those same key words along with words that indicate an expression will become part of an equation or inequality.

Addition	**Subtraction**	**Multiplication**	**Division**	**Equals**
Sum	Difference	Of	Quotient	Is
Increased by	Decreased by	Product	Ratio of	Equals
More than	Minus	Times	Percent	Will be
Combined	Less	Multiplied by	Fraction of	Gives
Together	Less than	Double	Out of	Yields
Total of	Fewer than	Twice	Per	Costs
Added to	Withdraws	Triple	Divided by	
Gained				
Raised				
Plus				

When taking a word problem and translating it to an equation or inequality, it is important to "**talk to the text**" or underline/highlight key phrases or words. By doing this it helps you see what is occurring in the problem.

Modeling Mathematics with Equations

A person's maximum heart rate is the highest rate, in beats per minute, that the person's heart should reach. One method to estimate the maximum heart rate states your age added to your maximum heart rate is 220. Using this method, write and solve an equation to find the maximum heart rate of a 15 year old.

Age	Added to	Maximum Heart Rate	Is	220

In the equation above, we did not know one of the quantities. When we do not know one of the quantities, we use a **variable** to represent the unknown quantity. When creating equations, it is important that whatever variable you use to represent the unknown quantity, you define or state what the variable represents.

Practice Examples: *In the examples below, "talk to the text" as you translate your word problems into equations. Define a variable to represent an unknown quantity, create your equation, and then solve your equation.*

1. Six less than four times a number is 18. What is the number?

Variables: ___________________________

Equation: ___________________________

2. You and three friends divide the proceeds of a garage sale equally. The garage sale earned $412. How much money did each friend receive?

Variables: ___________________________

Equation: ___________________________

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3. On her iPod, Mia has rock songs and dance songs. She currently has 14 rock songs. She has 48 songs in all. How many dance songs does she have?

Variables: ___________________________

Equation: ___________________________

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4. Brianna has saved $600 to buy a new TV. If the TV she wants costs $1800 and she saves $20 a week, how many months will it take her to buy the TV (4 weeks = 1 month)?

Variables: ___________________________

Equation: ___________________________

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5. It costs Raquel $5 in tolls to drive to work and back each day, plus she uses 3 gallons of gas. It costs her a total of $15.50 to drive to work and back each day. How much per gallon is Raquel paying for her gas?

Variables: ___________________________

Equation: ___________________________

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6. Mrs. Jackson earned a $500 bonus for signing a one year contract to work as a nurse. Her salary is $22 per hour. If her first week's check including the bonus is $1204, how many hours did Mrs. Jackson work?

Variables: ___________________________

Equation: ___________________________

7. Morgan subscribes to a website for processing her digital pictures. The subscription is $5.95 per month and 4 by 6 inch prints are $0.19. How many prints did Morgan purchase if the charge for January was $15.83?

Variables: ___________________________

Equation: ___________________________

8. A rectangle is 12m longer than it is wide. Its perimeter is 68m. Find its length and width (Hint: p = 2w + 2l).

Variables: ___________________________

Equation: ___________________________

9. The daycare center charges $120 for one week of care. Families with multiple children pay $95 for each additional child per week. Write an equation for the total cost for one week of care in terms of the number of children. How many children does a family have it they spend $405 a week in childcare?

Variables: ___________________________

Equation: ___________________________

10. The party store has a special on greeting cards. It charges $14 for 4 greeting cards and $1.50 for each additional card. Write an equation for the total cost of greeting cards in terms of the number of cards. What is the total cost for 9 greeting cards?

Variables: ___________________________

Equation: ___________________________

11. Clara has a coupon for $10 off her favorite clothing store. The coupon is applied before any discounts are taken. The store is having a sale and offering 15% off everything. If Clara has $50 to spend, how much can her purchases total before applying the discount to her coupon?

Variables: ___________________________

Equation: ___________________________

Day 7 – Creating Equations from a Context (Complex)

1. **Consider the following numbers:**
   45, 46, 47 102, 103, 104 30, 31, 32 99, 100, 101

a. What patterns do you notice?

b. How does the second number compare to the first number?

c. How does the third number compare to the first number?

2. **Consider the following numbers:**
   32, 34, 36 98, 100, 102 50, 52, 54 78, 80, 82

a. What patterns do you notice?

b. How does the second number compare to the first number?

c. How does the third number compare to the first number?

3. **Consider the following numbers:**
   45, 47, 49 103, 105, 107 29, 31, 33 157, 159, 161

a. What patterns do you notice?

b. How does the second number compare to the first number?

c. How does the third number compare to the first number?

Numbers that follow each other in order, without gaps, are called ____.

4. Create an expression for if you didn’t know the first number, but knew they were consecutive:

   a. Pattern in Problem 1: ____

   b. Pattern in Problem 2: ____

   c. Pattern in Problem 3: ____

Consecutive Numbers

Consecutive Numbers Chart				
Type of Consecutive Numbers	*Examples*	*Expressions for Terms* *First*	*Second*	*Third*
Consecutive Numbers	4, 5, 6 27, 28, 29	x	x + 1	x + 2
Consecutive Even Numbers	8, 10, 12 62, 64, 66	x	x + 2	x + 4
Consecutive Odd Numbers	23, 25, 27 89, 91, 93	x	x + 2	x + 4

1. The sum of three consecutive numbers is 72. What is the smallest of these numbers?

Variables: ___________________________________

Equation: ___________________________________

2. Find three consecutive odd integers whose sum is 261.

Variables: ___________________________________

Equation: ___________________________________