Name: ______________________________

Unit 5: Linear Functions

Learning Goal 5.2 – Contexts & Applications of Linear Functions

Learning Target #2: Applications of Linear Functions

- Identify the domain and range, x and y intercepts, intervals of increase and decrease, maximums and minimums, end behavior, and positive and negative areas from a graph
- Interpret linear functions in context
- Write an equation of a line given a point and slope or two points
- Analyze linear functions using different representations
- Find and interpret appropriate domains and ranges for authentic linear functions
- Calculate and interpret the average rate of change

Mon, 11/4 Day 10: Writing Equations of Lines	Tues, 11/5 No School	Wed, 11/6 Day 11: **5.1 Assessment;** Characteristics of Linear Functions	Thurs, 11/7 Day 12: Characteristics of Linear Functions	Fri, 11/8 Day 13: Standard Form
Mon, 11/11 Day 14: Standard vs Slope Intercept Form	Tues, 11/12 Day 15: Comparing Linear Functions	Wed, 11/13 Day 16: Comparing Linear Functions	Thurs, 11/14 Day 17: Remediation/Extra Day	Fri, 11/15 Day 18: **5.2 Assessment**
Mon, 11/18 Cumulative Exam Review	Tues, 11/19 **Cumulative Exam #2**	Wed. 11/20	Thurs, 11/21	Fri. 11/22

	Monday	Tuesday	Wednesday	Thursday	Friday
AM	**NONE**	**Mrs. Jackson 7:45 – 8:15 Room 1210**	**Mr. Webb 7:45 – 8:15 Room 1205**	**Mr. Watson 7:45 – 8:15 Room 1208**	**Mr. Watson 7:45 – 8:15 Room 1208**
PM	**Mrs. Petersen 3:30 – 4:30 Room 1210**	**Mr. Webb 3:30 – 4:30 Room 1205**	**NONE**	**Mrs. Jackson 3:30 – 4:30 Room 1210**	**NONE**

Day 11 – Characteristics of Linear Functions

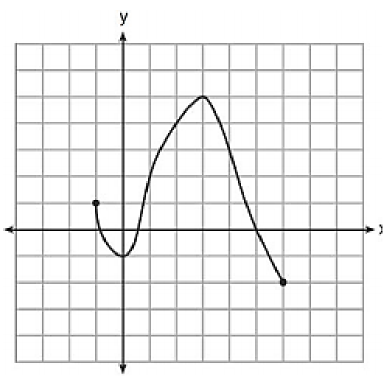
One key component to fully understanding linear functions is to be able to describe characteristics of the graph and its equation. **Important:** If a graph is a line (arrows), we need to assume that it goes on forever.

Domain and Range

Domain		
Define: All possible values of x	**Think:** How far left to right does the graph go?	**Write:** Smallest x ≤ x ≤ Biggest x *use < if the circles are open*
Range		
Define: All possible values of y	**Think:** How far down to how far up does the graph go?	**Write:** Smallest y ≤ y ≤ Biggest y *use < if the circles are open*

Non Linear Examples:

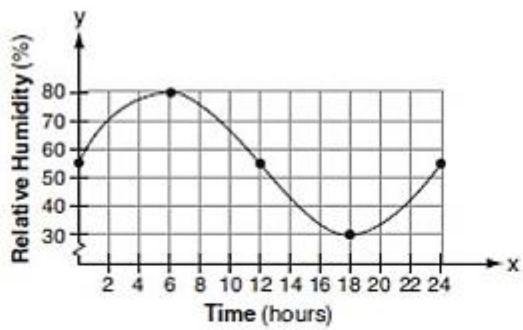
1.



Domain:

Range:

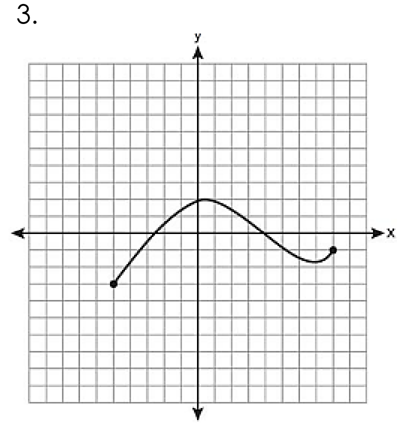
2.



Domain:

Range:

3.

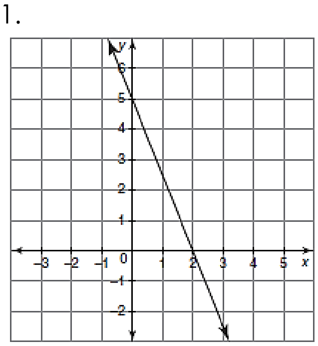


Domain:

Range:

Linear Examples:

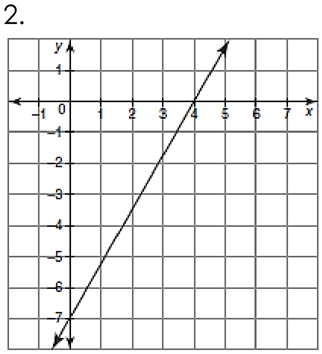
1.



Domain:

Range:

2.



Domain:

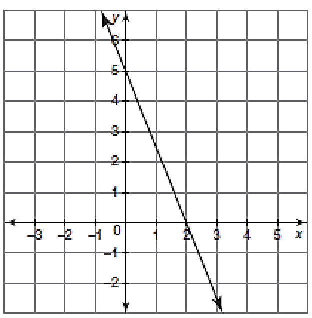
Range:

X and Y intercepts (including zeros)

Y-Intercept		
Define: Point where the graph crosses the y-axis	**Think:** At what coordinate point does the graph cross the y-axis?	**Write:** (0, b)
X-Intercept		
Define: Point where the graph crosses the x-axis	**Think:** At what coordinate point does the graph cross the x-axis?	**Write:** (a, 0)
Zero		
Define: Where the function (y-value) equals 0	**Think:** At what x-value does the graph cross the x-axis?	**Write:** x = ____

Linear Examples:

1.

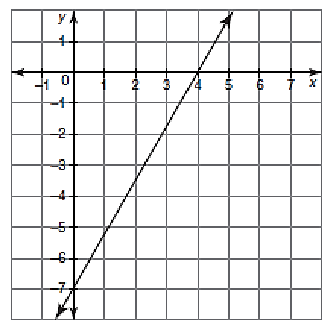


Y-intercept:

X-intercept

Zero:

2.

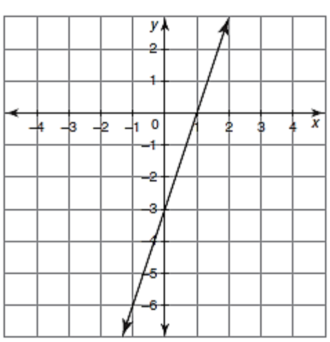


Y-intercept:

X-intercept:

Zero:

3.

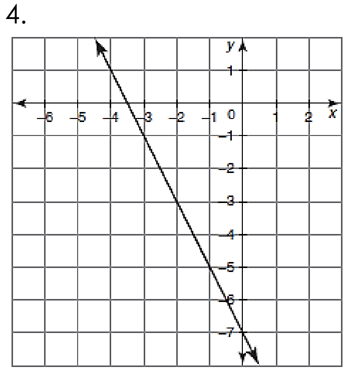


Y-intercept:

X-intercept

Zero:

4.



Y-intercept:

X-intercept:

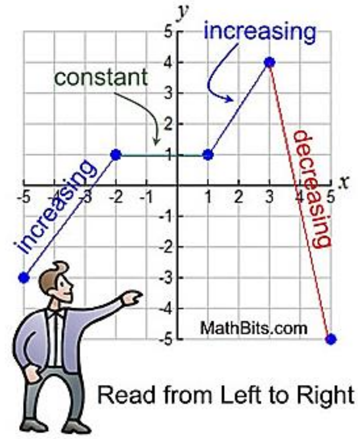
Zero:

Interval of Increase and Decrease

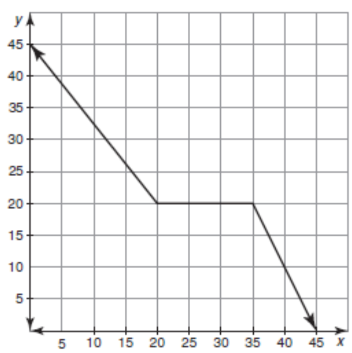
Interval of Increase		
Define: The part of the graph that is rising as you read left to right.	**Think:** From left to right, is my graph going up?	**Write:** x value where it starts increasing < x < x value where it stops increasing

Interval of Decrease		
Define: The part of the graph that is falling as you read from left to right.	**Think:** From left to right, is my graph going down?	**Write:** x value where it starts decreasing < x < x value where it stops decreasing

Interval of Constant		
Define: The part of the graph that is a horizontal line as you read from left to right.	**Think:** From left to right, is my graph a flat line?	**Write:** x value where it starts flat-lining < x < x value where it stops flat-lining



Non Linear Example:



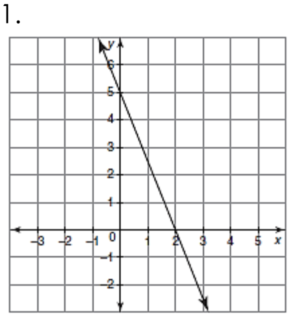
Interval of Increase:

Interval of Decrease:

Interval of Constant:

Linear Examples:

1.

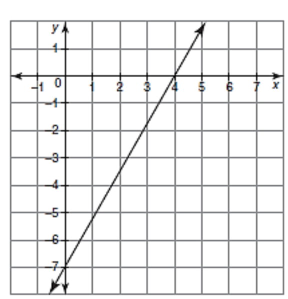


Interval of Increase:

Interval of Decrease:

Interval of Constant:

2.



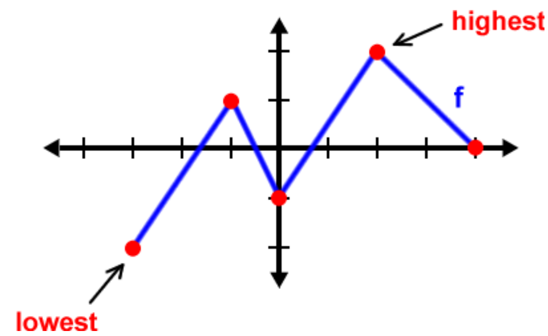
Interval of Increase:

Interval of Decrease:

Interval of Constant:

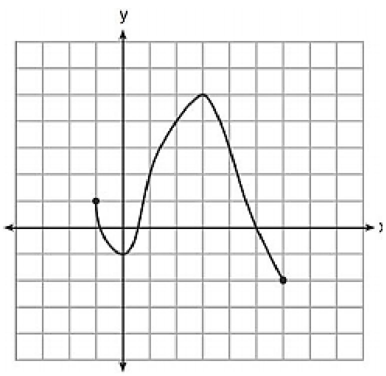
Maximum and Minimum (Extrema)

Maximum		
Define: Highest point or peak of a function.	**Think:** What is my highest point or value on my graph?	**Write:** If none, write none Otherwise, y = biggest y-value
Minimum		
Define: Lowest point or valley of a function.	**Think:** What is the lowest point or value on my graph?	**Write:** If none, write none Otherwise, y = smallest y-value



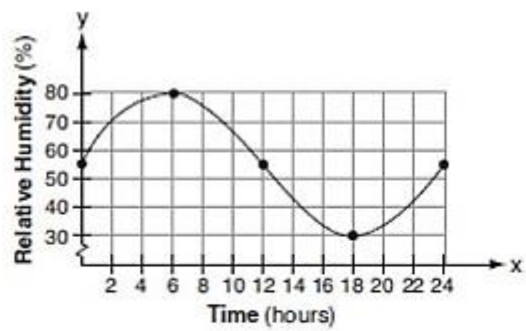
Non Linear Examples:

1.



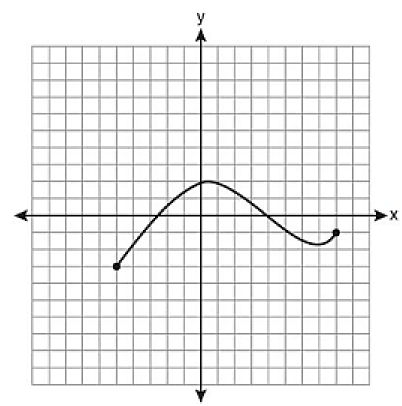
Maximum:
Minimum:

2.



Maximum:
Minimum:

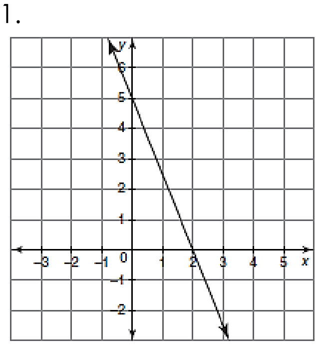
3.



Maximum:
Minimum:

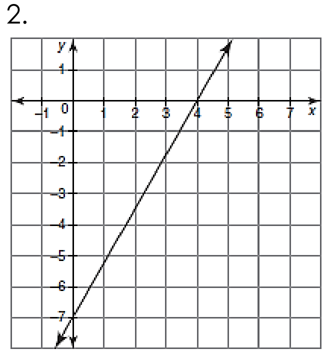
Linear Examples:

1.



Maximum:
Minimum:

2.

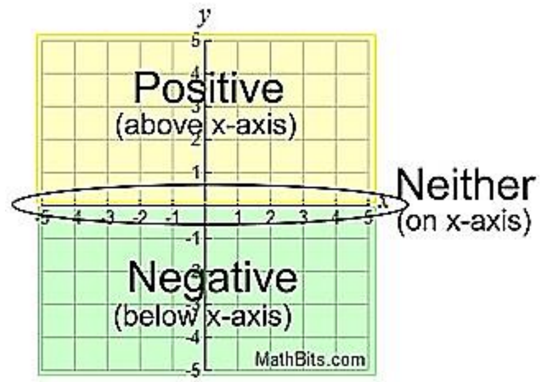


Maximum:
Minimum:

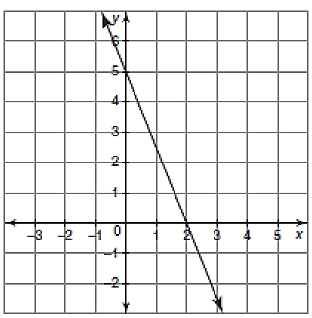
Day 12 – Characteristics of Linear Functions (cont'd)

Positive and Negative Regions on a Graph

Positive		
Define: The part of the function that is above the x-axis.	**Think:** Which part of the function is in the positive region and where?	**Write:** Inequality using zero value (x)
Negative		
Define: The part of the function that is below the x-axis.	**Think:** Which part of the function is in the negative region and where?	**Write:** Inequality using zero value (x)



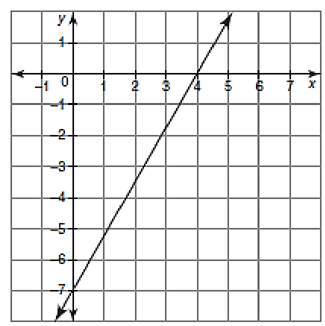
1.



Positive: ______________________

Negative: ______________________

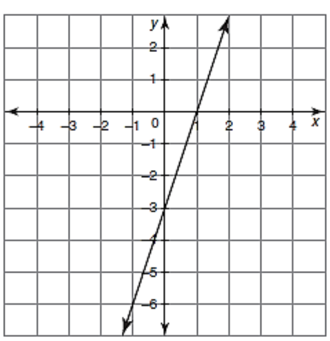
2.



Positive: ______________________

Negative: ______________________

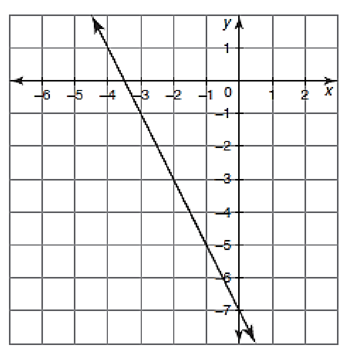
3.



Positive: ______________________

Negative: ______________________

4.

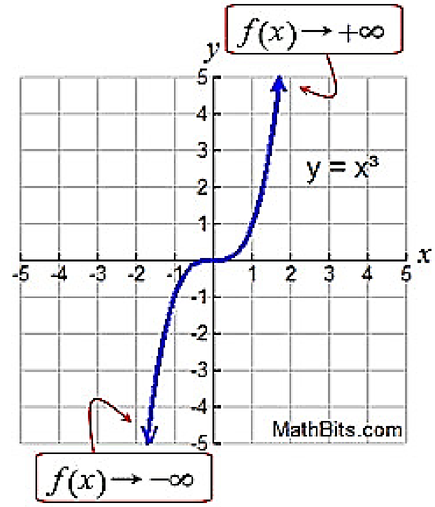


Positive: ______________________

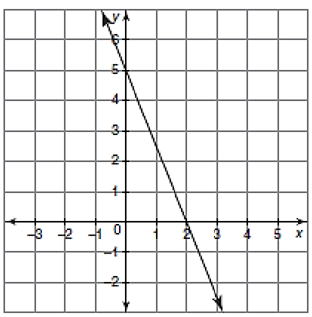
Negative: ______________________

End Behavior

<table>
<tr><th colspan="2">End Behavior</th></tr>
<tr><td colspan="2">Define:
Behavior of the ends of the function (what happens to the y-values or f(x)) as x approaches positive or negative infinity. The arrows indicate the function goes on forever so we want to know where those ends go.</td></tr>
<tr><td>Think:
As x goes to the left (negative infinity), what direction does the left arrow go?</td><td>Write:
As x $\rightarrow -\infty$, f(x) $\rightarrow$ _____</td></tr>
<tr><td>Think:
As x goes to the right (positive infinity), what direction does the right arrow go?</td><td>Write:
As x $\rightarrow \infty$, f(x) $\rightarrow$ _____</td></tr>
</table>



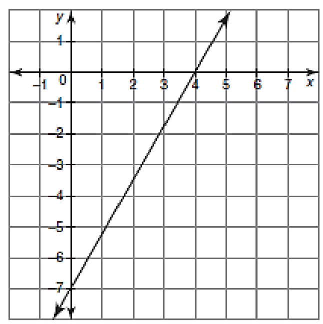
1.



As x $\rightarrow -\infty$, f(x) $\rightarrow$ _____

As x $\rightarrow \infty$, f(x) $\rightarrow$ _____

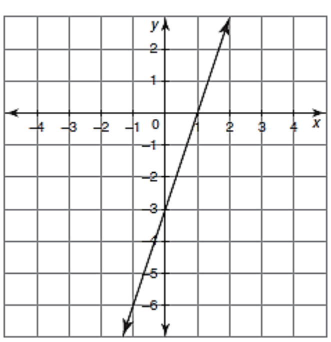
2.



As x $\rightarrow -\infty$, f(x) $\rightarrow$ _____

As x $\rightarrow \infty$, f(x) $\rightarrow$ _____

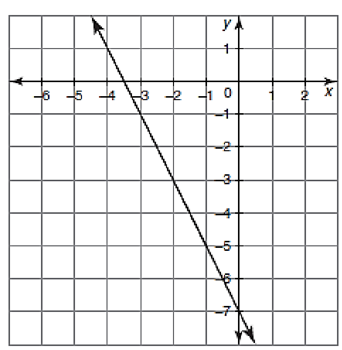
3.



As x $\rightarrow -\infty$, f(x) $\rightarrow$ _____

As x $\rightarrow \infty$, f(x) $\rightarrow$ _____

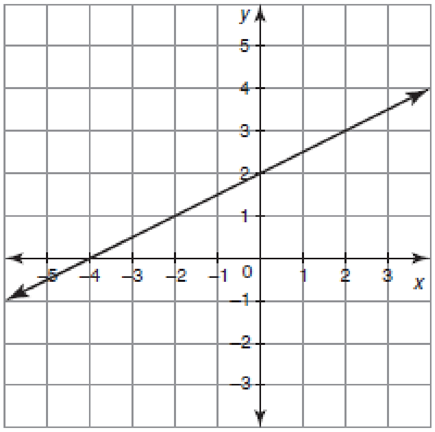
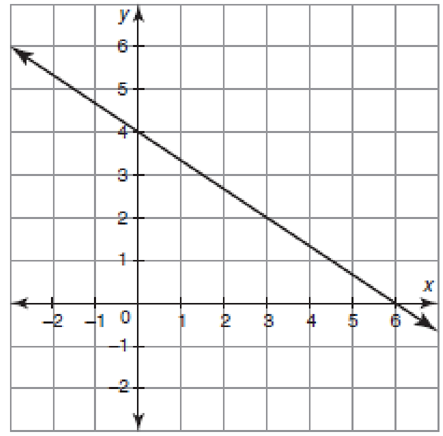
4.



As x $\rightarrow -\infty$, f(x) $\rightarrow$ _____

As x $\rightarrow \infty$, f(x) $\rightarrow$ _____

Practice

Practice Example 1	Practice Example 2
	
Domain: *Range:*	*Domain:* *Range:*
Y-intercept: *X-intercept:* *Zero:*	*Y-intercept:* *X-intercept:* *Zero:*
Interval of Increase: *Interval of Decrease:* *Interval of Constant:*	*Interval of Increase:* *Interval of Decrease:* *Interval of Constant:*
Maximum: *Minimum:*	*Maximum:* *Minimum:*
Positive: *Negative:*	*Positive:* *Negative:*
End Behavior: As x → -∞, f(x) → ______ As x → ∞, f(x) → ______	*End Behavior:* As x → -∞, f(x) → ______ As x → ∞, f(x) → ______

Day 13 – Characteristics of Linear Functions (Real World)

Now that you have learned all the characteristics that apply to linear functions, we are going to focus on a few characteristics that have very real world applications to them – slope, domain & range, and intercepts.

The Real Number System

When we apply domain and range to real world situations, we need to consider what types of numbers are suitable for a domain and range. Typically, we describe domain and range using one of the types of number classifications.

Types of Numbers	Example
Counting Numbers	1, 2, 3, 4… (Zero is not included)
Whole Numbers	0, 1, 2, 3… (Also called non-negative integers)
Integers	…-3, -2, -1, 0, 1, 2, 3, …
Rational Numbers	Everything above plus decimals & fractions
Real Numbers	Everything above plus irrational numbers

Most of the real world applications of domain and range do not include rational numbers (you can't have a fractional piece of an item or person) or non-negative numbers (such as time).

Domain & Range

When determining appropriate domains and ranges for a function, think about what the independent and dependent quantities are and what type of numbers are appropriate and which are not appropriate.

Example 1: A plumber charges $96 an hour for making house calls to do plumbing work. What would be an appropriate domain and range? Assume he charges by hour.

Independent Quantity: Dependent Quantity:

Domain: Range:

Example 2: Laura is selling cookies to raise funds for a school club. Each cookie costs $0.50. What would be an appropriate domain and range?

Independent Quantity: Dependent Quantity:

Domain: Range:

Example 3: Rentals cars at ABC Rental Car Company cost $100 to rent, plus $1 per mile. What would be an appropriate domain and range?

Independent Quantity: Dependent Quantity:

Domain: Range:

Example 4: Jason goes to an amusement park where he pays $8 admission and $2 per ride. He has $30 to spend.

Independent Quantity: Dependent Quantity:

Domain: Range:

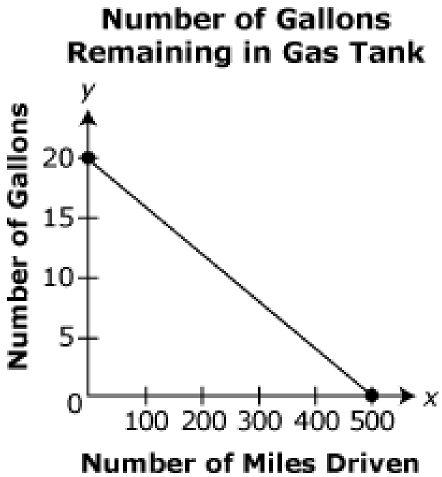
Example 5: Hunter is shopping for pencils. He has $5.00 from his allowance and he finds the pencils he wants cost $0.65 each.

Independent Quantity: Dependent Quantity:

Domain: Range:

Intercepts

1. *A car owner recorded the number of gallons of gas remaining in the car's gas tank after driving a number of miles. Use the graph below to answer the following questions.*

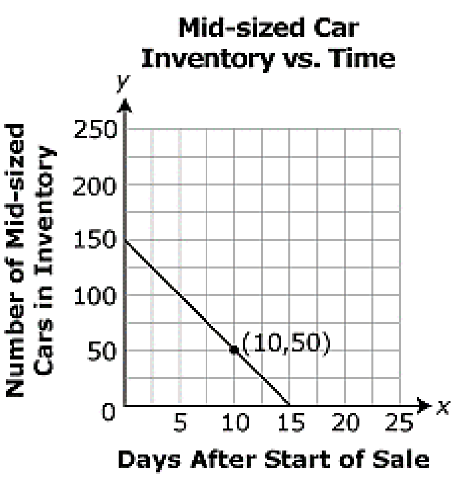


a. What does x-intercept represent on the graph?

b. What does the y-intercept represent on the graph?

c. What does the point (200, 12) represent on the graph?

2. *The graph below shows the relationship between the number of mid-sized cars in a car dealer's inventory and the number of days after the start of a sale.*



a. What does x-intercept represent on the graph?

b. What does the y-intercept represent on the graph?

c. What does the point (10, 50) represent on the graph?
Is the point a solution of the graph?

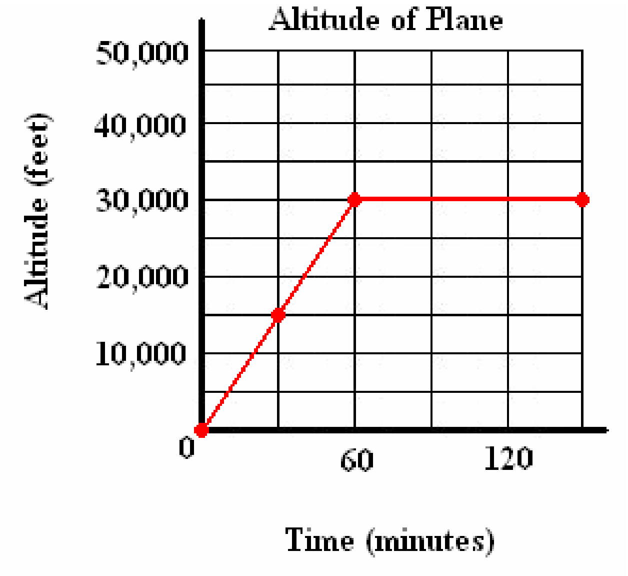
d. What does the point (5, 125) represent on the graph?
Is the point a solution of the graph?

Slope/Average Rate of Change

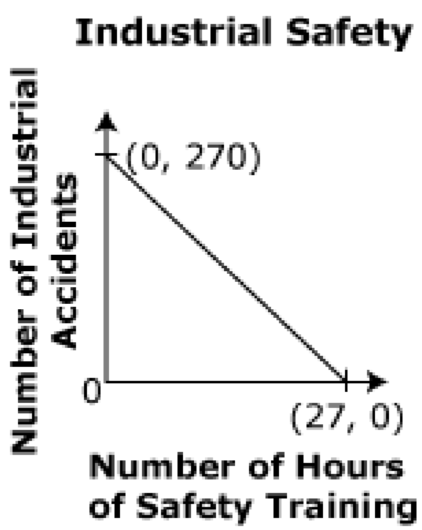
Example 1: The graph shows the altitude of a plane.

a. Find the plane's rate of change during the first hour.

b. Find the plane's rate of change during the second hour.



Example 2: An industrial-safety study finds there is a relationship between the number of industrial accidents and the number of hours of safety training for employees. This relationship is shown in the graph below.



a. Find the rate of change.

b. Explain what it represents.

Applications of Slope Intercept Form

Y	=	M	X	+	B
Output		Slope	Input		Y-intercept (0, b)
Dependent Variable		Rate	Independent Variable		Starting Amount One Time Fee
Range		$\frac{\text{change in y}}{\text{change in x}}$	Domain		Beginning

When a problem involves a ***constant rate or speed and a beginning amount****, it can be written using slope intercept form. You need to recognize which value is the slope and which is the y-intercept.*

Example 1: An airplane 30,000 feet above the ground begins descending at a rate of 2000 feet per minute. Assume the plane continues at the same rate of descent. The plan's height and minutes above the ground are related to each. What is the altitude after 5 minutes?

Independent Quantity:

Dependent Quantity:

Slope:

Y-intercept:

Equation:

Example 2: Suppose you receive $100 for a graduation present and you deposit it into a savings account. Then each week after that, you add $20 to your savings account. When will you have $460?

Independent Quantity:

Dependent Quantity:

Slope:

Y-intercept:

Equation:

Day 14 – Standard Form of Equations

Scenario: In the mid 1800's, delivering mail and news across the American Great Plains was time consuming and made for a long delay in getting vital information from side of the country to the other. At the time, most mail and news traveled by stagecoach along the main stagecoach lines at about 8 miles per hour. The Pony Express Riders averaged about 10.7 miles per hour. The long stretch of 782 miles from the two largest cities on either side of the plains, St. Louis and Denver, was a very important part of this trail.

a. Use the variable x to write an expression to represent the distance the stagecoach was driven in miles.	b. Use the variable y to write an expression to represent the distance the Pony Express rode in miles.	c. Write an expression for the distance that was traveled using both of these methods on one trip.
8x	**10.7y**	**8x + 10.7y**
d. Write an equation that represents using both methods to deliver mail from St. Louis to Denver. **8x + 10.7y = 782**		

a. If the Pony Express Riders rode for 20 hours from St. Louis before handing off the mail to a stagecoach, how long would it take the stagecoach to get to Denver?

X	Y

b. If the stagecoach rode for 50 hours from St. Louis before handing off the mail to a Pony Express Rider, how long would it take the rider to get to Denver?

X	Y

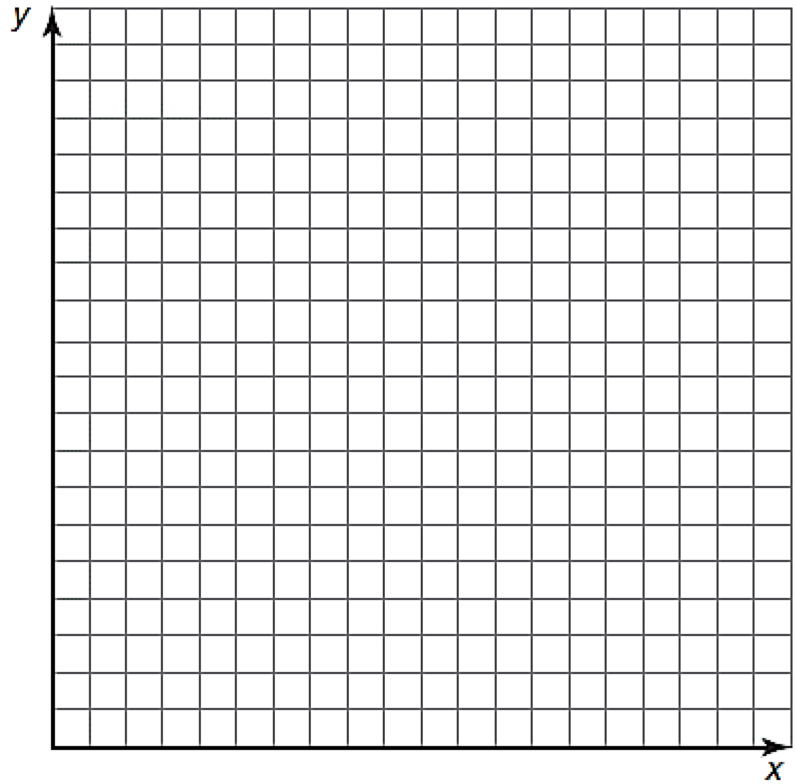
c. If mail was delivered by stagecoach only, how long would it take the stagecoach to get the mail from St. Louis to Denver?

X	Y

d. If mail was delivered by Pony Express Riders only, how long would it take a rider to get the mail from St. Louis to Denver?

X	Y

Time the mail was in a Stagecoach (hours)	Time the mail was with the Pony Express (hours)
	20
50	
	0
0	



The Parts of the Pony Express Problem

The equation, 8x + 10.7y = 782 is in **standard form of a linear equation**, which is **Ax + By = C**. Below, describe what each variable or expression represents in this equation.

X	
Y	
8x	
10.7y	
8x + 10.7y	
782	
x-intercept	
y-intercept	

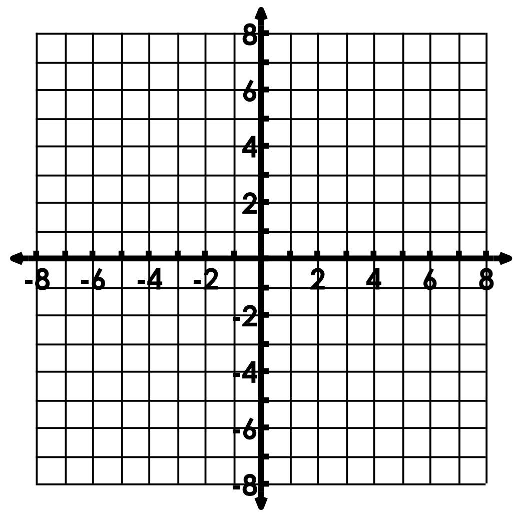
Finding x & y intercepts

X –intercepts	**Y-intercepts**
Written as (a, 0)	Written as (0, b)
The value of the y-coordinate is always 0.	The value of the x-coordinate is always 0.

Practice: Find the x and y intercepts of each equation. Then graph.

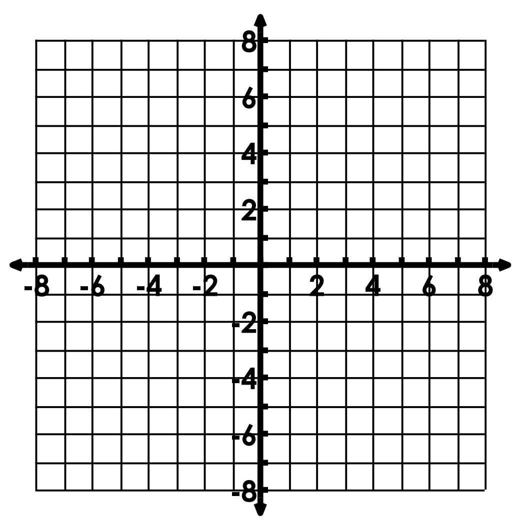
a. $2x - 5y = 10$

x-intercept: y-intercept:



b. $3x + 6y = -18$

x-intercept: y-intercept:



Day 15: Comparing Standard Form and Slope Intercept Form

	Standard Form	Slope Intercept Form
Form	*Ax + By = C* a, b, and c are constants	*y = mx + b* m = slope b = y-intercept
Information	Gives x intercept (when substituting 0 for y) Gives y-intercept (when substituting 0 for x)	Gives slope and y-intercept
Advantages	Easy to calculate x and y intercepts Helpful when we solve systems of equations (Unit 3) using elimination	Easily determine slope and y-intercept Easiest and fastest to graph the line Only form you can put in the graphing calculator
Disadvantages	Do not know the slope unless you convert to slope intercept form (solve for y) A, B, and C do not stand for anything obvious (like slope or y-intercept) Harder to graph a line	Finding the x-intercept takes a little more work Not every linear equation can be written in slope intercept form (like x = 5)
Context	Adding or subtracting two amounts and setting equal to a total *Example: Tickets for the school play cost $5.00 for students and $8.00 for adults. On opening night $1600 was collected in ticket sales.* *5x + 8y = 1600*	Multiplying a constant to a changing amount and then adding or subtracting a starting amount *Example: Carl has $200 in his bank account and each week he withdraws $25 dollars.* *y = 200 – 25x*

Practice with Standard and Slope Intercept Form in a Context

Practice: For each scenario, create an equation and solve for the missing variable.

a. A bookstore has mystery novels on sale for $2 each and sci-fi novels on sale for $3 each. Bailey has $30 to spend on books. How many mystery novels can she buy if she buys 6 sci-fi novels?

b. Your little brother is having a party at the local zoo. The zoo charges a party fee of $50 plus $5 for each guest. How many guests did he invite if the total cost was $115?

c. Alex's goal is to sell $100 worth of tickets to the school play. The tickets are $4 for students and $10 for adults. How many student tickets does he need to sell if he sells 6 adult tickets?

d. It costs $4 to order a chicken sandwich and $3 to order a cheeseburger form the local fast food restaurant down the street for dinner for the math team before their competition. They have $60 to spend on food. Calculate the x and y intercepts of this problem and interpret your answers in terms of the problem.

Day 16 – Comparing Linear Functions

Linear Functions can come in many forms:

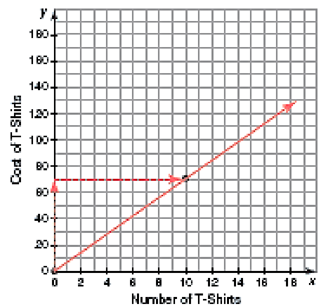
Context:

The basketball team won the championship. They are selling special championship T-shirts for a cost of $7 each.

rate of change

y-intercept = 0

Graph:



$\frac{\text{rise}}{\text{run}} = \frac{\$7}{1\text{ T-shirt}}$

Table:

Number of T-shirts	Cost in Dollars
0	0
1	7
2	14
3	21
4	28
5	35

rate of change $= \frac{7-0}{1-0} = \frac{\$7}{\text{T-shirt}}$

y-intercept

Equation:

Let *y* represent: total cost of T-shirts

Let *x* represent: number of T-shirts

$y = 7x$

$y = 7x + 0$

Rate of change y-intercept

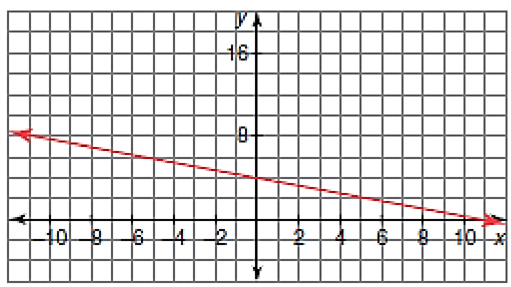
Now that you have studied linear functions and their characteristics for over two weeks, you need to be able to compare and answer questions in whatever form is given to you. The best way to develop your comparing skills is just to practice; there is not actual lesson – just practice problems for you to try.

Practice 1: *Which function has the biggest y-intercept?*

Function A:

x	y
0	6
1	9
2	12
3	15

Function B:

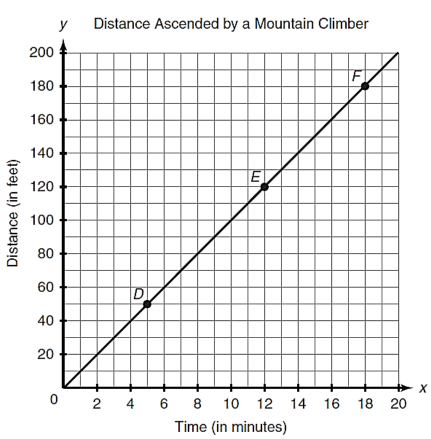


Function C:

$y = -10x + 2.5$

Practice 2: *Which function has the greatest rate of change?*

Function A:



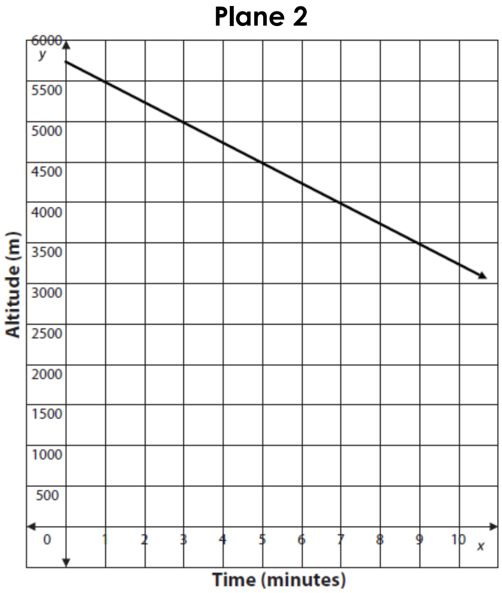
Function B:

Number of Minutes on an Exercise Bike	Total Number of Calories Burned
15	180
30	360
45	540
60	720

Function C:

$30x + 2y = -24$

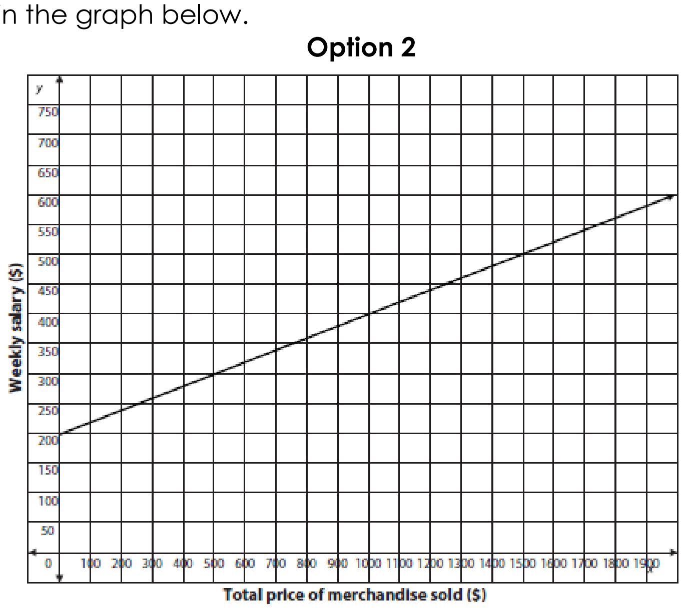
Practice 3: Two airplanes are in flight. The function f(x) = 400x + 1200 represents the altitude, f(x), of Plane 1 after x minutes. The graph below represents the altitude of the second airplane.



Compare the starting altitudes of the two planes.

Compare the rate of change of the two planes.

Practice 4: Your employer has offered two pay scales for you to choose from. The first option is to receive a base salary of $250 a week plus 15% of the price of any merchandise you sell. The second option is represented in the graph below.



a. Create an equation to represent the first option for one week's worth of pay.

b. Create an equation to represent the second option for one week's worth of pay.

c. Which option has a higher base salary? Explain how you know.

d. Which option has a higher rate for selling merchandise? Explain how you know.