Graphing Quadratics in Vertex Form

Vertex Form of a Quadratic Function:

$$y = a(x - h)^2 + k$$

a determines how the graph opens

positive **a**, graph opens ____________

negative **a**, graph opens ____________

(_____ , _____) is our vertex.

NOTE: Our vertex is at (h, k), **NOT** (-h, k).

Identifying the Vertex Practice

Find the vertex of the following:

1) $y = (x - 18)^2 + 9$ Vertex = (_____ , _____)

2) $y = 4(x + 6)^2 - 7$ Vertex = (_____ , _____)

3) $y = (x - 2)^2 - 2$ Vertex = (_____ , _____)

Find the vertex for each of the following quadratics and determine whether the graph opens up or down:

a) $y = (x - 1)^2 - 2$ Vertex = (_____ , _____) Graph Opens ___________ because a is ___

b) $y = -3(x + 4)^2 + 1$ Vertex = (_____ , _____) Graph Opens ___________ because a is ___

c) $y = 2x^2 + 3$ Vertex = (_____ , _____) Graph Opens ___________ because a is ___

d) $y = -(x - 3)^2$ Vertex = (_____ , _____) Graph Opens ____________ because a is ___

Steps for Graphing in Vertex Form

1) Find the vertex (h, k).

2) Use your vertex as the center for your table and determine two x values to the left and right of your h value and substitute those x values back into the equation to determine the y values.

    - Using practice problem number 3, let's practice filling in our table.
      $y = (x - 2)^2 - 2$

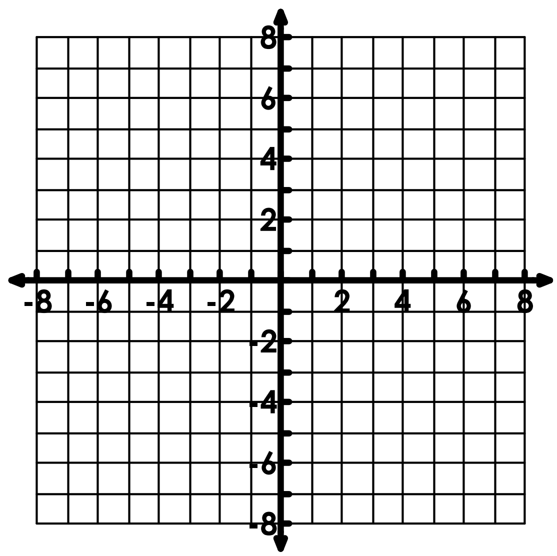
x					
y					

3) Plot your points and connect them from left to right!

Graphing in Vertex Form Examples

Example 1: Graph $y = (x - 1)^2 - 2$.

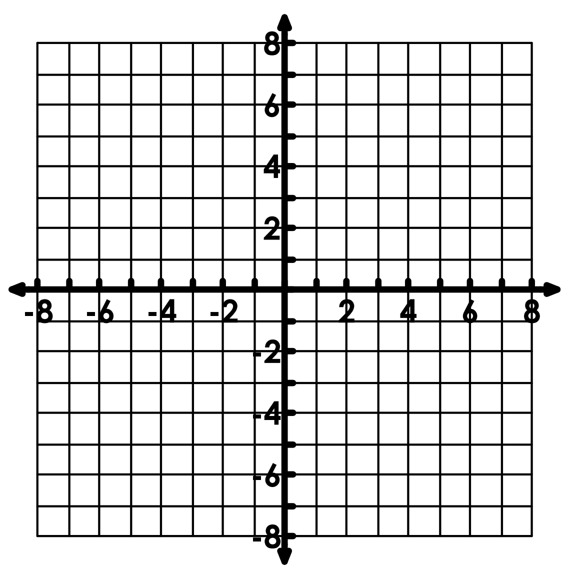
Vertex = (_____ , _____)



x	-1	0		2	3
y					

Example 2: Graph: $y = -3(x + 4)^2 + 1$.

Vertex = (_____ , _____)

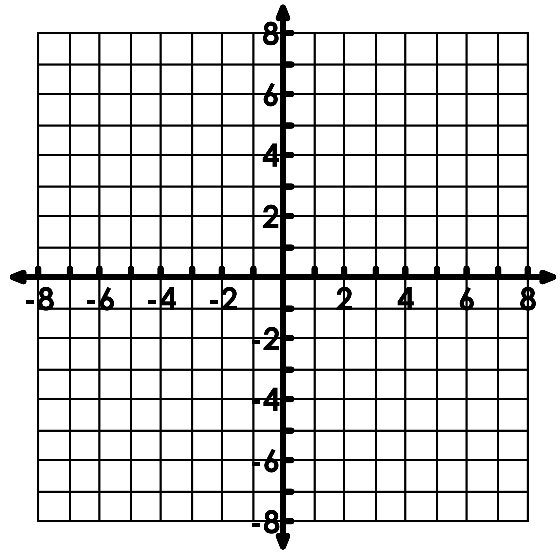


x					
y		-2			-11

Example 3: Graph $y = 2x^2 + 3$.

Vertex = (_____ , _____)

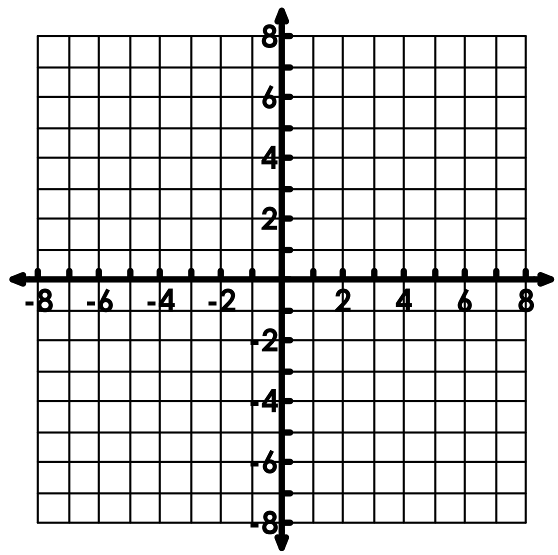
x					
y					



Example 4: Graph: $y = -(x - 3)^2$.

Vertex = (_____ , _____)

x					
y					



Using a Graphing Calculator to Graph Quadratics in Vertex Form

Use a graphing calculator to graph our last example problem, example 4: $y = -(x - 3)^2$

1. Hit **Y =** and enter the equation into y_1.

2. Hit **Graph** (Hit **Zoom,** then **6** to get back to a standard viewing window, if necessary).

3. You can also use the table on the graphing calculator to compare to your table and note the symmetry along the vertex. Hit **2nd** followed by **Graph** (you really want the Table feature). Scroll through the table until you find where the y_1 values stop decreasing and begin increasing, the point it switches at is our vertex.

Characteristics of Quadratics

One key component to fully understanding quadratic functions is to be able to describe characteristics of the graph and its equation.

Domain and Range

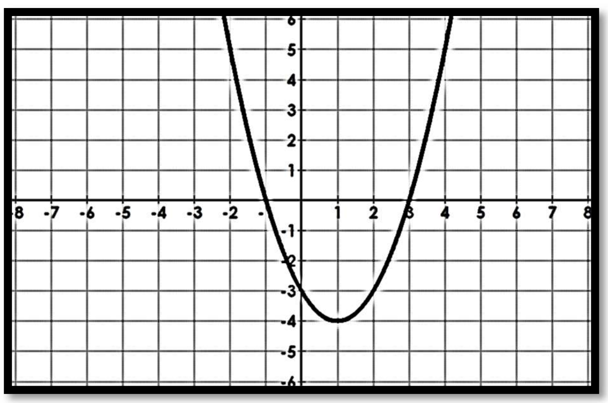
Domain

Define:	Think:	Write:
All possible values of x	How far left to right does the graph go?	Smallest x ≤ x ≤ Biggest x *use < if the circles are open*

Range

Define:	Think:	Write:
All possible values of y	How far down to how far up does the graph go?	y ≤ highest y value (opens down) y ≥ lowest y value (opens up)

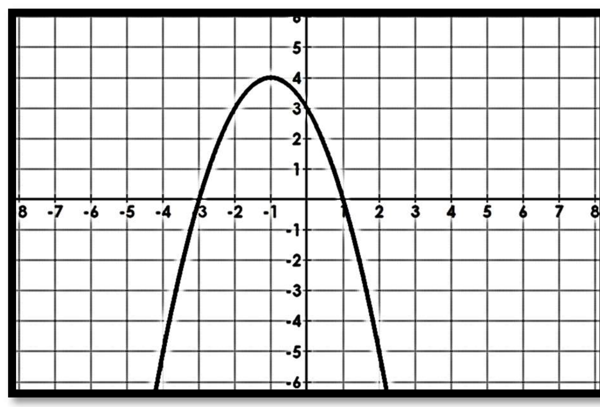
Graph 1



Domain:

Range:

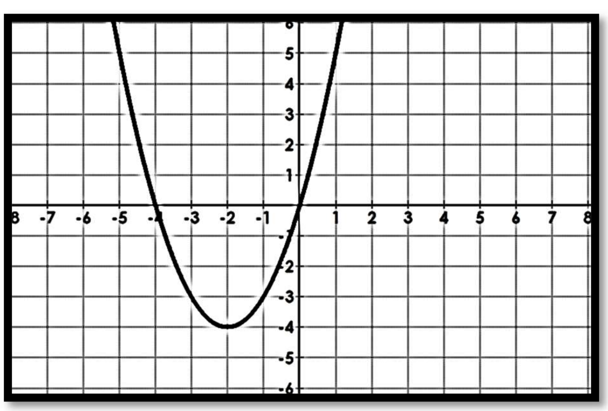
Graph 2



Domain:

Range:

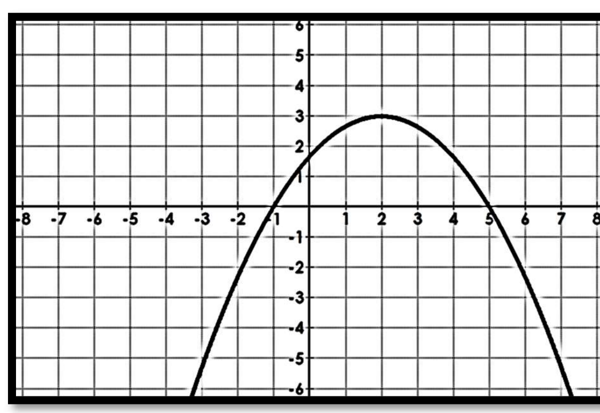
Graph 3



Domain:

Range:

Graph 4



Domain:

Range:

Zeros and Intercepts

Y-Intercept

Define:	Think:	Write:
Point where the graph crosses the y-axis	At what coordinate point does the graph cross the y-axis?	(0, b)

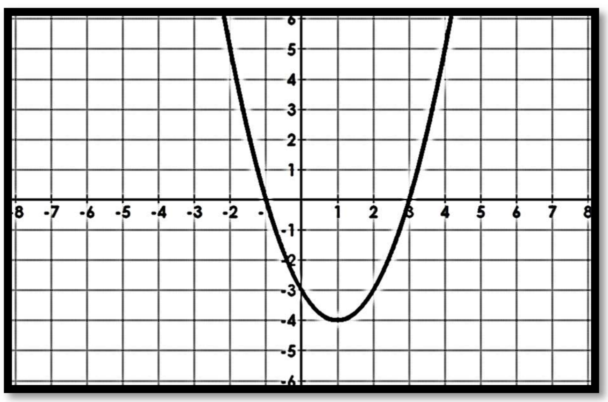
X-Intercept

Define:	Think:	Write:
Point where the graph crosses the x-axis	At what coordinate point does the graph cross the x-axis?	(a, 0)

Zero

Define:	Think:	Write:
Where the function (y-value) equals 0	At what x-value does the graph cross the x-axis?	x = ____

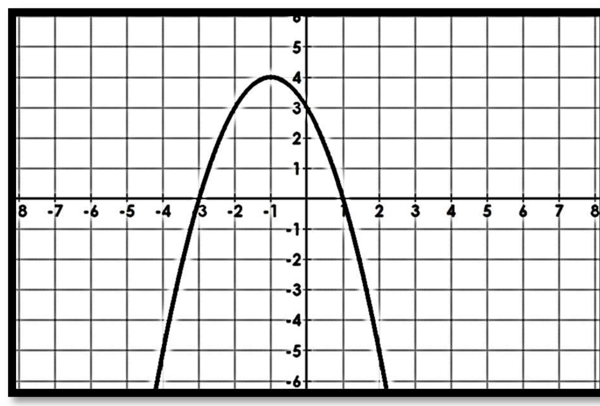
Graph 1



X-intercepts: Y-intercept:

Zeros:

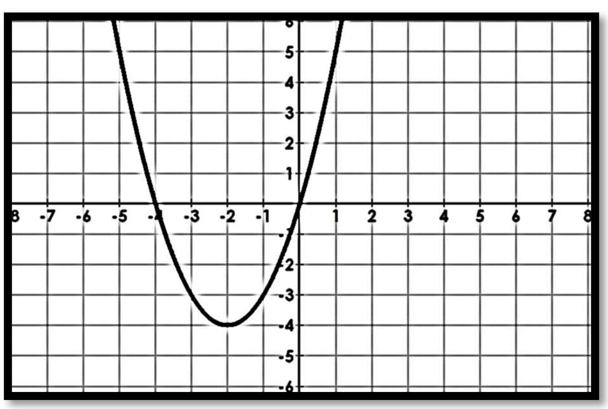
Graph 2



X-intercepts: Y-intercept:

Zeros:

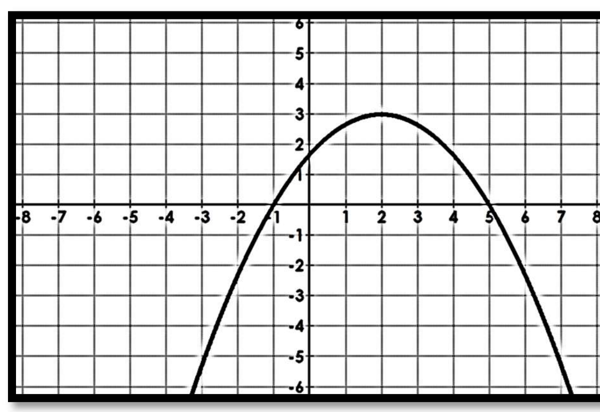
Graph 3



X-intercepts: Y-intercept:

Zeros:

Graph 4



X-intercepts: Y-intercept:

Zeros:

Vertex & Axis of Symmetry

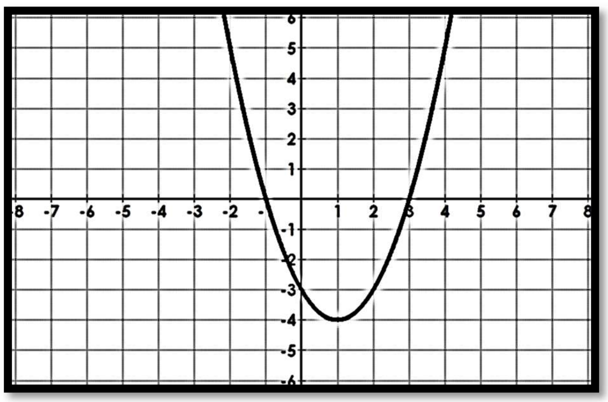
Vertex

Define:	Think:	Write:
Highest or lowest point or peak of a parabola	What is my highest or lowest point on my graph?	Name the point (h, k)

Axis of Symmetry

Define:	Think:	Write:
The vertical line that divides the parabola into mirror images and runs through the vertex	What imaginary, vertical line would make the parabola symmetrical?	x = h (x value of the vertex)

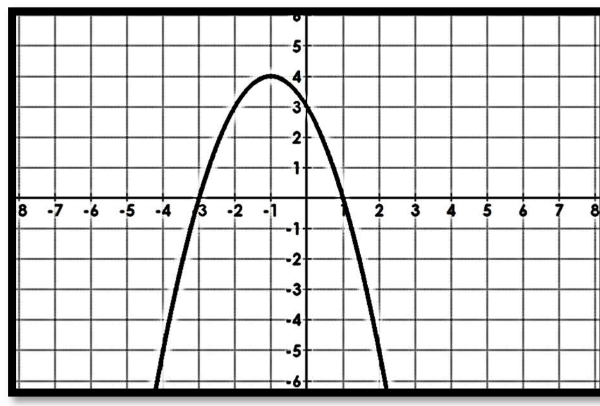
Graph 1



Vertex:

Axis of Symmetry:

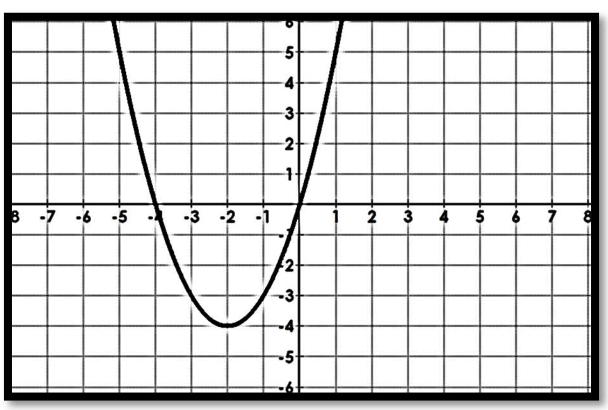
Graph 2



Vertex:

Axis of Symmetry:

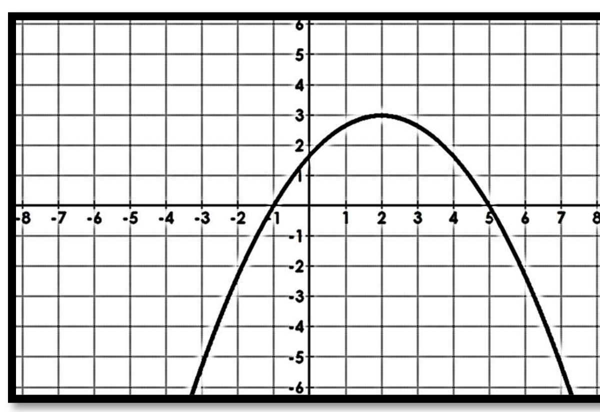
Graph 3



Vertex:

Axis of Symmetry:

Graph 4



Vertex:

Axis of Symmetry:

Extrema

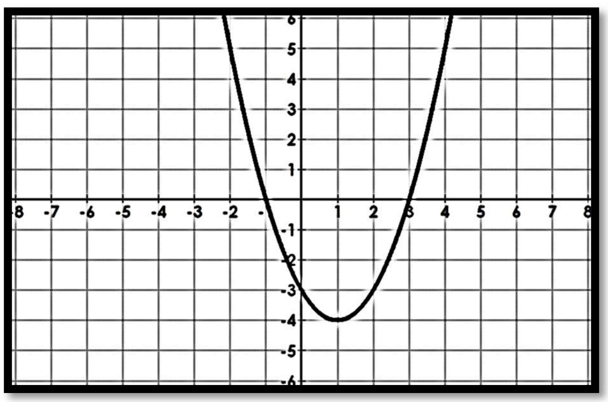
Maximum

Define:	**Think:**	**Write:**
Highest point or peak of a function.	What is my highest point on my graph?	y = k (y-value of the vertex)

Minimum

Define:	**Think:**	**Write:**
Lowest point or valley of a function.	What is the lowest point on my graph?	y = k (y-value of the vertex)

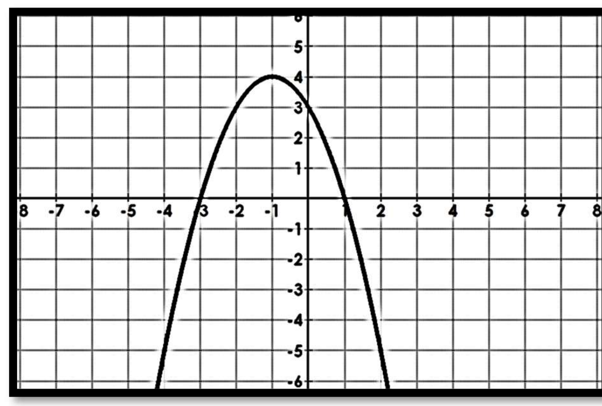
Graph 1



Extrema:

Min/Max Value:

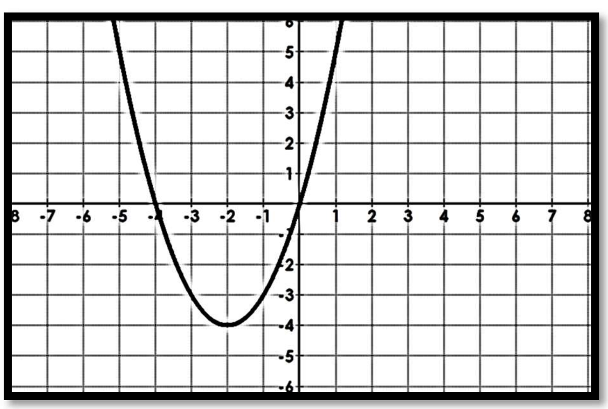
Graph 2



Extrema:

Min/Max Value:

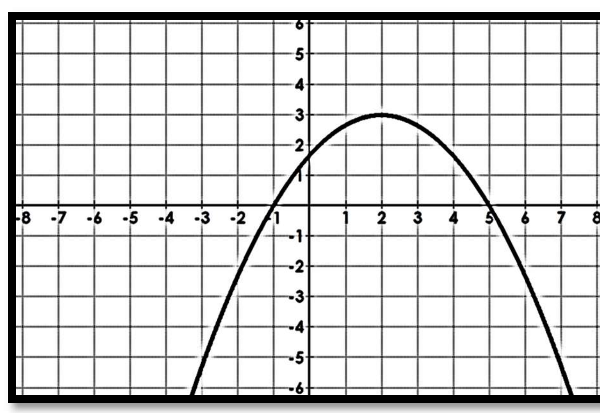
Graph 3



Extrema:

Min/Max Value:

Graph 4



Extrema:

Min/Max Value:

End Behavior

End Behavior

Define:

Behavior of the ends of the function (what happens to the y-values or f(x)) as x approaches positive or negative infinity. The arrows indicate the function goes on forever so we want to know where those ends go.

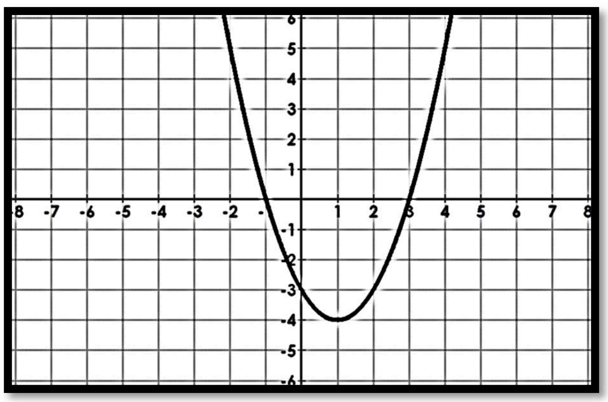
Think:
As x goes to the left (negative infinity), what direction does the left arrow go?

Write:
As x → -∞, f(x) → _____

Think:
As x goes to the right (positive infinity), what direction does the right arrow go?

Write:
As x → ∞, f(x) → _____

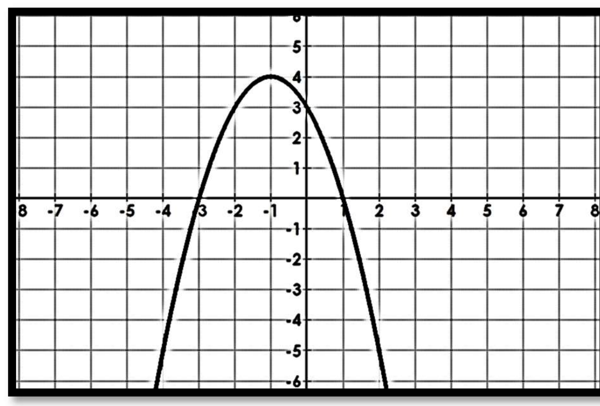
Graph 1



As x → -∞, f(x) → _______.

As x → ∞, f(x) →_______.

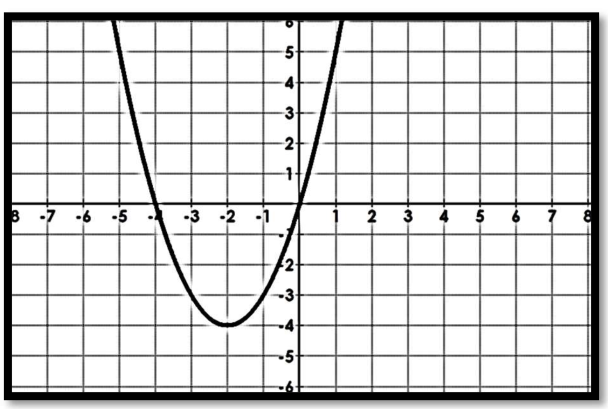
Graph 2



As x → -∞, f(x) → _______.

As x → ∞, f(x) →_______.

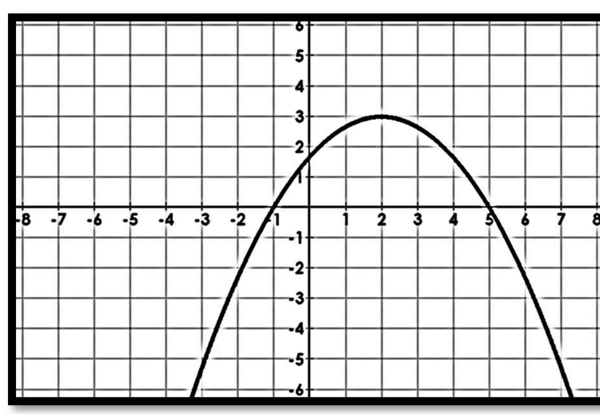
Graph 3



As x → -∞, f(x) → _______.

As x → ∞, f(x) →_______.

Graph 4



As x → -∞, f(x) → _______.

As x → ∞, f(x) →_______.

Intervals of Increase and Decrease

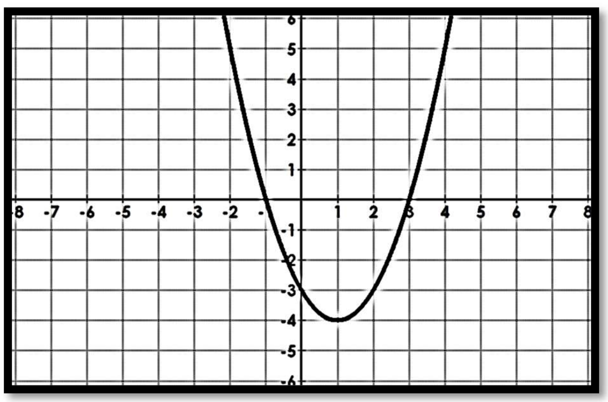
Interval of Increase

Define:	Think:	Write:
The part of the graph that is rising as you read left to right.	From left to right, is my graph going up?	An inequality using the x-value of the vertex

Interval of Decrease

Define:	Think:	Write:
The part of the graph that is falling as you read from left to right.	From left to right, is my graph going down?	An inequality using the x-value of the vertex

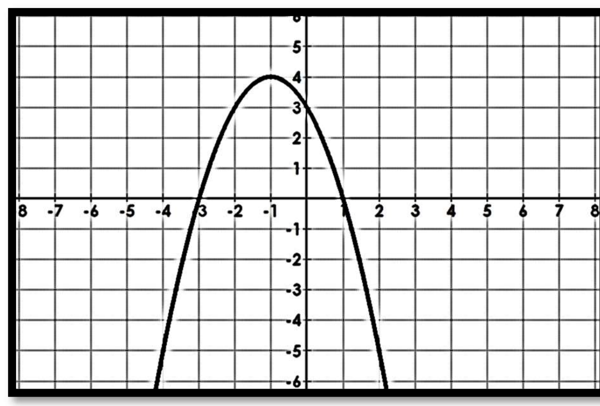
Graph 1



Interval of Increase:

Interval of Decrease:

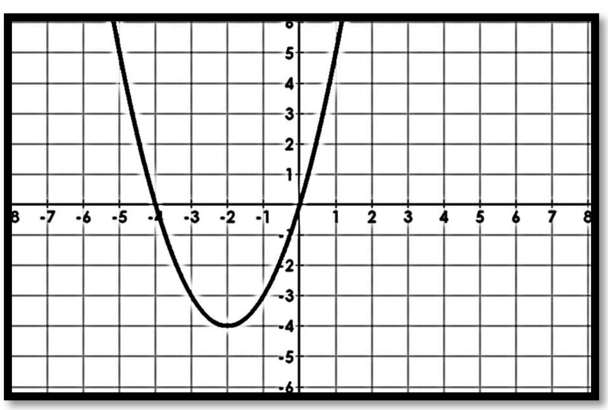
Graph 2



Interval of Increase:

Interval of Decrease:

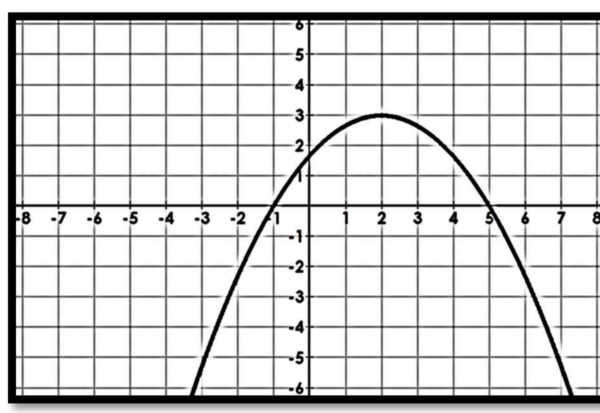
Graph 3



Interval of Increase:

Interval of Decrease:

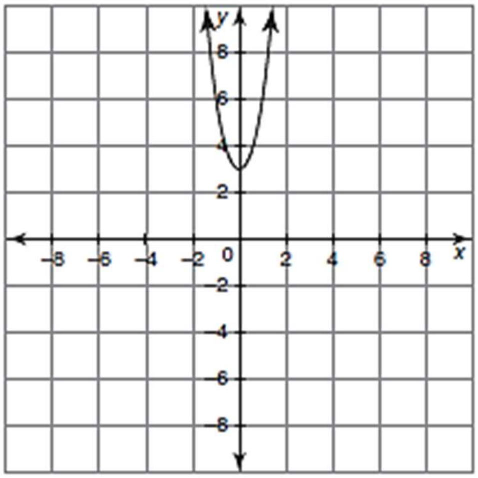
Graph 4



Interval of Increase:

Interval of Decrease:

Practice: Describe the characteristics of the following graphs:



Domain: ____________________ **Range:** ______________________

Vertex: ____________________ **Axis of Sym.** __________________

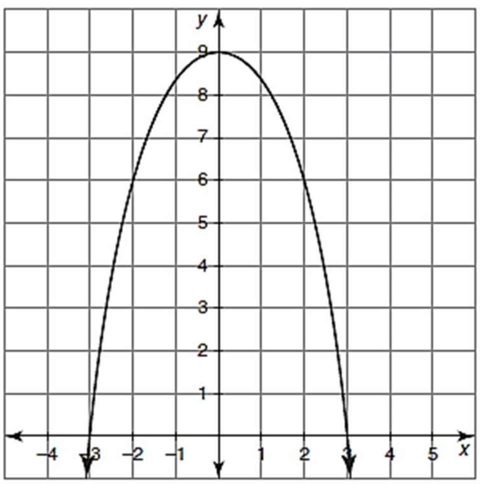
Y-Intercept: ________________ **Zeroes:** ______________________

Extrema: ____________________ **Max/Min Value:** _____________

Int of Inc: __________________ **Int of Dec:** ___________________

Positive: ____________________ **Negative:** _____________________

End Behavior: As x $\rightarrow$ -∞, f(x) $\rightarrow$ ______. As x $\rightarrow$ ∞, f(x) $\rightarrow$______



Domain: ____________________ **Range:** ______________________

Vertex: ____________________ **Axis of Sym.** __________________

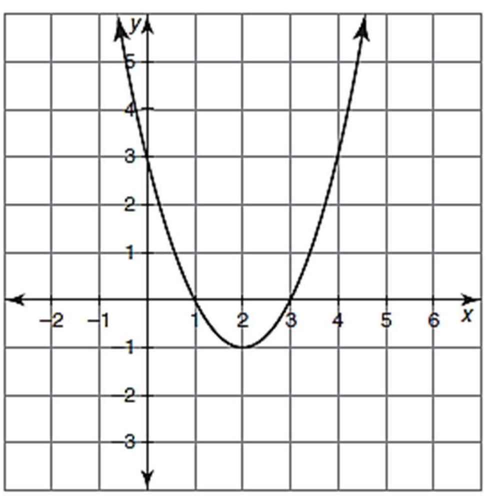
Y-Intercept: ________________ **Zeroes:** ______________________

Extrema: ____________________ **Max/Min Value:** _____________

Int of Inc: __________________ **Int of Dec:** ___________________

Positive: ____________________ **Negative:** _____________________

End Behavior: As x $\rightarrow$ -∞, f(x) $\rightarrow$ ______. As x $\rightarrow$ ∞, f(x) $\rightarrow$______



Domain: ____________________ **Range:** ______________________

Vertex: ____________________ **Axis of Sym.** __________________

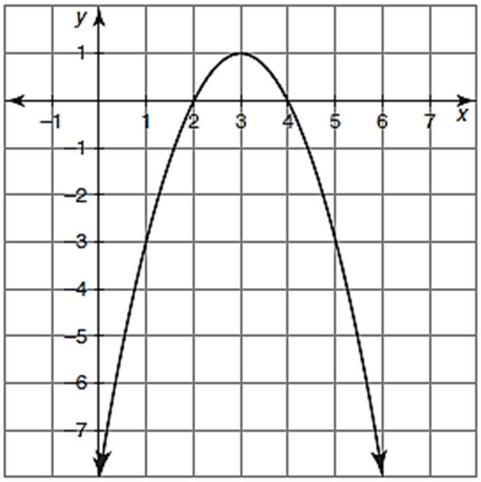
Y-Intercept: ________________ **Zeroes:** ______________________

Extrema: ____________________ **Max/Min Value:** _____________

Int of Inc: __________________ **Int of Dec:** ___________________

Positive: ____________________ **Negative:** _____________________

End Behavior: As x $\rightarrow$ -∞, f(x) $\rightarrow$ ______. As x $\rightarrow$ ∞, f(x) $\rightarrow$______



Domain: ____________________ **Range:** ______________________

Vertex: _____________________ **Axis of Sym.**__________________

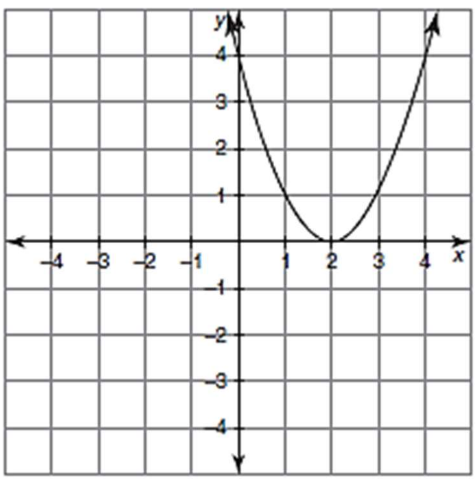
Y-Intercept: ________________ **Zeroes:** ______________________

Extrema: ____________________ **Max/Min Value:** _____________

Int of Inc: __________________ **Int of Dec:** ___________________

Positive: _____________________ **Negative:** ____________________

End Behavior: As x → -∞, f(x) → ______. As x → ∞, f(x) →______



Domain: ____________________ **Range:** ______________________

Vertex: _____________________ **Axis of Sym.**__________________

Y-Intercept: ________________ **Zeroes:** ______________________

Extrema: ____________________ **Max/Min Value:** _____________

Int of Inc: __________________ **Int of Dec:** ___________________

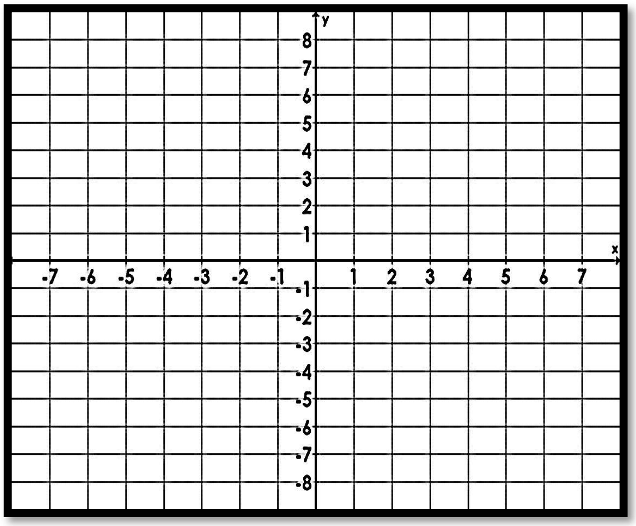
Positive: _____________________ **Negative:** ____________________

End Behavior: As x → -∞, f(x) → ______. As x → ∞, f(x) →______

Quadratic Transformations

The **parent function** of a function is the simplest form of a function. The parent function for a quadratic function is **$y = x^2$** or **$f(x) = x^2$**. Graph the parent function below.

x	x^2
-3	
-2	
-1	
0	
1	
2	
3	



The U-shaped graph of a quadratic function is called a

____________________________.

The highest or lowest point on a parabola is called the

____________________________.

One other characteristic of a quadratic equation is that one of the terms is always ______________________.

There are several different forms a quadratic function can be written in, but the one we are going to work with for today is called **vertex form**. In the following explorations below, you are going to learn the effect of a, h, and k values have on the parent graph.

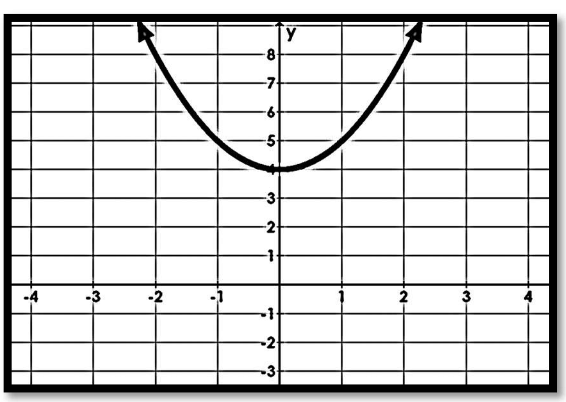
Vertex Form

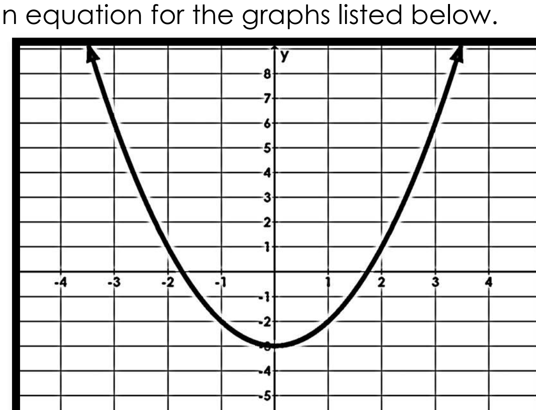
$$f(x) = a(x-h)^2 + k$$

Vertex: ____________

The K Value ~ $y = x^2 + k$

Describe the transformations and name the vertex. Create an equation for the graphs listed below.



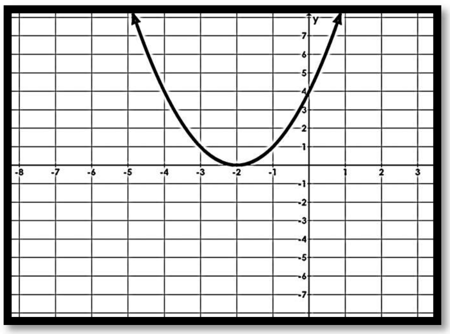


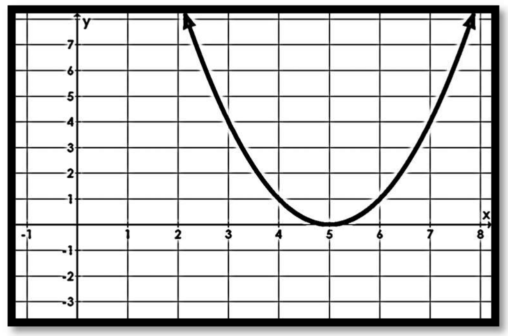
Given the transformations listed below, create an equation that would represent the transformations.

1. Shifted up 8 units
2. Shifted up 20 units
3. Shifted down 5 units

The H Value ~ $y = (x - h)^2$

Describe the transformations and name the vertex. Create an equation for the graphs listed below.





Given the transformations listed below, create an equation that would represent the transformations.

1. Shifted right 8 units
2. Shifted left 20 units
3. Shifted left 5 units

Putting It All Together with H and K

Practice: Identify the transformations and vertex from the equations below.

Equation	Transformations	Vertex
1. $y = (x - 2)^2 + 4$		
2. $y = (x + 3)^2 - 2$		
3. $y = (x - 9)^2 - 5$		
4. $y = (x + 5)^2 + 6$		

The A Value~ $y = ax^2$

Describe the transformations from the given function to the transformed function.

a. $f(x) = x^2 \rightarrow f(x) = 4x^2$

b. $y = x^2 \rightarrow y = \frac{1}{4}x^2$

c. $f(x) \rightarrow 6\,f(x)$

d. $f(x) = x^2 \rightarrow f(x) = -x^2$

f. $y = x^2 \rightarrow y = -\frac{1}{2}x^2$

g. $f(x) \rightarrow -4f(x)$

Putting It All Together with A, H, and K

Practice: Given the equations below, name the vertex and describe the transformations:

Equation	Transformations	Vertex
1. $y = -(x - 4)^2 + 7$		
2. $y = -2(x + 2)^2 + 5$		
3. $y = \frac{1}{2}(x - 3)^2 - 8$		

Practice: Create an equation to represents the following transformations:

a. Shifted down 4 units, right 1 unit, and reflected across the x-axis

b. Shifted up 6 units, reflected across the x-axis, and stretch by a factor of 3

c. Shifted up 2 units, left 4 units, reflected across the x-axis, and shrunk by a factor of ¾.