## Unit 9: Solving Quadratic Equations

After completion of this unit, you will be able to...

## Learning Target \#9.1: Solving Quadratic Equations

- Solve a quadratic equation by factoring a GCF.
- Solve a quadratic equation by factoring when a is not 1 .
- Create a quadratic equation given a graph or the zeros of a function.
- Solve a quadratic equation by finding square roots.
- Solve a quadratic equation by completing the square.
- Solve a quadratic equation by using the Quadratic Formula.
- Solve a quadratic equation by analyzing the equation and determining the best method for solving.
- Solve application problems using quadratic equations.

| Monday | Tuesday | Wednesday | Thursday | Friday |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \hline \mathbf{2 / 1 0} \\ \text { Day } 1 \\ \text { Solving by Factoring } \end{gathered}$ | $\begin{gathered} \hline \text { 2/11 } \\ \text { Day } 2 \\ \text { Solving by Factoring } \end{gathered}$ | $2 / 12$ <br> Day 3 <br> Solving by Factoring | $2 / 13$ <br> Review for Cumulative Exam | $2 / 14$ <br> Cumulative Exam <br> (Unit 7 \& 8) |
| $2 / 17$ <br> Winter Break | $2 / 18$ <br> Winter Break | $2 / 19$ <br> Winter Break | $2 / 20$ <br> Winter Break | $2 / 21$ <br> Winter Break |
| $2 / 24$ <br> Day 3 <br> Solving by Factoring Review | $2 / 25$ <br> Day 4 <br> Solving by Square Roots | $2 / 26$ <br> Day 5 <br> Solving by Square Roots | $2 / 27$ <br> Day 6 <br> Solving by Completing the Square | 2/28 <br> Day 7 <br> Finding the Vertex via Completing the Square |
| $3 / 2$ <br> Day 8 <br> Solving by Quadratic Formula | 3/3 <br> Day 9 <br> Solving by Quadratic Formula | 3/4 <br> Day 10 <br> Quadratic Formula Applications | $3 / 5$ <br> Day 11 <br> Determining Best <br> Method and Review Day | 3/6 <br> Day 12 <br> 9.1 Learning Assessment |

Tutoring Times

|  | Monday | Tuesday | Wednesday | Thursday | Friday |
| :---: | :---: | :---: | :---: | :---: | :---: |
| AM | Mrs. Jackson 7.45-8.15 Room 1210 | Mr. Phillips 7.45-8.15 Room 1206 | Mrs. Jackson \& Mr. Webb 7.45-8.15 Room 1210 Room 1205 | Mr. Watson \& Mr. Phillips 7.45-8:15 Room 1208 Room 1206 | Mr. Watson 7.45-8.15 Room 1208 |
| PM | NONE | Mrs. Petersen 3:30-4:30 Room 1210 | NONE | NONE | NONE |

## Day 1 - 3: Solving by Factoring

Take a look at the following graph. Do you know what type of graph it is? List some of the things you see:


## The Main Characteristics of a Quadratic Function

- A quadratic function always has an exponent of $\qquad$ . Therefore, a quadratic function always has the term of $\qquad$ _.
- The standard form of a quadratic equation is $\qquad$
- The U-shaped graph is called a $\qquad$ .
- The highest or lowest point on the graph is called the $\qquad$ .
- The points where the graph crosses the x-axis are called the $\qquad$ .
- The points where the graph crosses are also called the $\qquad$ to the quadratic equation. A quadratic equation can have $\qquad$ , $\qquad$ or $\qquad$ solutions.

In this unit, we are going to explore how to solve quadratic equations. The solutions to the quadratic equations can look very different depending on what the graph of the quadratic equation looks like. We are going to explore what we learned in Unit 7 (factoring) and how we can apply that to solving quadratic equations.

## Exploration with Factoring and Quadratic Graphs

Find the zeros of the following graphs and then factor the expressions on the right:

1. $y=x^{2}-6 x+5$

2. $y=x^{2}+3 x$

3. $y=x^{2}-8 x+12$

4. $y=2 x^{2}+x-6$

a. What do you notice about the zeros on the graph and the factored form of your equation?
b. What is the value of $y$ when the parabola crosses the $x$-axis for each graph?
c. If you were to replace the $y$ in your equation, with your answer in part b, how do you think you could solve your equations so your answers match the zeros on your graph?

## Zero Product Property and Factored Form

A polynomial or function is in factored form if it is written as the product of two or more linear binomial factors. The zero product property is used to solve an equation when one side is zero and the other side is a product of binomial factors.
Examples: a. $(x-2)(x+4)=0$
b. $x(x+4)=0$
c. $(x+3)^{2}=0$

Practice: Identify the zeros of the functions:
a. $y=(x+4)(x+3)$
b. $f(x)=(x-7)(x+5)$
c. $y=x(x-9)$
d. $f(x)=5(x-4)(x+8)$

Using the zero product property, go back and solve your factored equations for their zeros on page 3.
Practice: Create an equation to represent the following graphs:

$x=$ $\qquad$ \& $\qquad$

$$
y=
$$

$\qquad$

$x=$ $\qquad$ \& $\qquad$

$$
y=
$$

$\qquad$

In this unit, you will be solving quadratic equations. In order to understand what we mean by "solving" quadratic equations, you must understand exactly what we will be solving for from an equation.

## Solving a quadratic equation really means:

The place(s) where the graph crosses the x-axis has several names. They can be referred as:

## Review of Factoring Types

When you factor, remember to always check for a GCF first!

| Factoring A = 1 | Difference of Two Squares |
| :---: | :---: |
| Factor: $x^{2}+3 x-18$ | Factor: $x^{2}-16$ |
|  |  |
| Factoring A not 1 |  |
| Factor: $2 x^{2}-13 x+15$ | Factoring by GCF |
| Factor: $x^{2}-6 x$ |  |
| Factor: $3 x^{2}-3 x-60$ |  |

## Practice with Solving Quadratic Equations by Factoring

1. $y=(x+2)(2 x-1)$
2. $y=(x-3)(x-1)$

Factored Form: $\qquad$ Factored Form: $\qquad$
Zeros: $\qquad$ Zeros: $\qquad$
3. $y=x^{2}-14 x+48$
4. $y=x^{2}-6 x+9$

Factored Form: $\qquad$
Zeros: $\qquad$
Factored Form: $\qquad$

Zeros: $\qquad$
5. $x^{2}+6 x+8=0$
6. $5 x=x^{2}-6$

Factored Form: $\qquad$ Factored Form: $\qquad$
Zeroes: $\qquad$ Zeroes: $\qquad$
7. $-x^{2}=2 x+1$
8. $y=x^{2}-9$
$\qquad$ Factored Form: $\qquad$

Zeroes: $\qquad$ Zeroes: $\qquad$
9. $x^{2}+4 x=32$

Factored Form: $\qquad$
Zeroes: $\qquad$
11. $x^{2}+7 x=0$

Factored Form: $\qquad$
Zeros: $\qquad$
13. $y=5 x^{2}+14 x-3$

Factored Form: $\qquad$

Zeroes: $\qquad$
12. $2 x^{2}-6 x=0$

Factored Form: $\qquad$

Zeros: $\qquad$
14. $2 x^{2}-8 x-42=0$

Factored Form: $\qquad$
Zeros: $\qquad$

Factored Form: $\qquad$
Zeros: $\qquad$
15. $16 x^{2}-8 x=0$

Factored Form: $\qquad$
Zeroes: $\qquad$
17. $y=4 x^{2}-22 x+10$

Factored Form: $\qquad$
Zeroes: $\qquad$
18. $-3 x^{2}-12 x=0$

Factored Form: $\qquad$
Zeroes: $\qquad$

Factored Form: $\qquad$

Zeroes: $\qquad$

## Day 4: Solving by Finding Square Roots

Review: If possible, simplify the following radicals completely.
a. $\sqrt{25}$
b. $\sqrt{125}$
c. $\sqrt{24}$

Explore: Solve the following equations for x :
a. $x^{2}=16$
b. $x^{2}=4$
C. $x^{2}=9$
d. $x^{2}=1$

What operation did you perform to solve for $x$ ?

How many of you only had one number as an answer for each equation?
Well, let's take a look at the graph of this function.


After looking at the graph, what values of $x$ produce a $y$ value of $1,4,9$ and 16 ?

What would be your new answers for the previous equations?
a. $x^{2}=16$
b. $x^{2}=4$
C. $x^{2}=9$
d. $x^{2}=1$

In order to be successful at today's lesson, you need to understand two things: how to solve a linear equation and understand that square roots and squares are inverses of each other.

Key Idea \#1: Solving a Linear Equation:
S/A D/M E P = SADMEP

Practice: Solve the following equations for $x$ :
a. $2 x+8=12$
b. $3(x+5)=6$
C. $10 x+9=499$

Practice: Take the following numbers and square them; then take the square root of your new number to show how you end up with the number you started with.


## Solving by Taking Square Roots without Parentheses



Solve the following for x :

1) $x^{2}=49$
2) $x^{2}=20$
3) $x^{2}=7$
4) $3 x^{2}=108$
5) $2 x^{2}=128$
6) $x^{2}-11=14$
7) $7 x^{2}-6=57$
8) $2 x^{2}+8=170$
9) $x^{2}=0$
10) $10 x^{2}+9=499$
11) $4 x^{2}-6=74$
12) $3 x^{2}+7=301$

## Applications of Solving by Square Roots

## Falling Objects:

$h=-16 t^{2}+h_{0} \quad h_{0}=$ starting height, $h=$ ending height

1. The tallest building in the USA is in Chicago, Illinois. It is 1450 ft tall. How long would it take a penny to drop from the top of the building to the ground?
2. When an object is dropped from a height of 72 feet, how long does it take the object to hit the ground?

Application:
3. For a period of 48 months, the average monthly operating costs for a small business $C$ (in dollars) is approximated by the model $C=0.55 t^{2}+550$, where $\dagger$ is the number of months. During which month was the average operating cost $\$ 1430$ ?

## Day 5 - Solving by Finding Square Roots (More Complicated)



Solve the following for $\mathbf{x}$ :

1) $(x-4)^{2}=81$
2) $(p-4)^{2}=16$
3) $10(x-7)^{2}=440$
4) $\frac{1}{2}(x+8)^{2}=14$
5) $-2(x+3)^{2}-16=-48$
6) $3(x-4)^{2}+7=67$

## Solving Literal Equations with Quadratic Equations

A literal equation is an equation with more than one variable. When solving literal equations, you would use the properties of equality to isolate the variable you are solving for and treat the other variables like constants.

1. Solve for $s$ : $A=6 s^{2}$
2. Solve for $r: \quad A=\pi r^{2}$
3. Solve for $r: A=\frac{\pi r^{2} s}{360}$
4. Solve for $b: a^{2}+b^{2}=c^{2}$.
5. Solve for $\mathrm{C}: \mathrm{E}=\mathrm{mc}^{2}$
6. Solve for $s: V=\frac{1}{3} s^{2} h$
7. The formula for finding the volume of a square pyramid is $V=1 / 3 s^{2} h$, where $s$ represents the side length of the square base and $h$ represents the height. What equation could be used to find the height of the square pyramid?
8. The volume of a box with a square bottom and a height of 4 in is given by $V(x)=4 x^{2}$, where $x$ is the length (in inches) of the sides of the bottom of the box.

a. If the volume of the box is $289 \mathrm{in}^{3}$, find the dimensions of the box.
b. Are there two possible answers to part (a)? Why or why not?

## Day 6 - Solving by Completing the Square

Some trinomials form special patterns that can easily allow you to factor the quadratic equation. We will look at two special cases:
Review: Factor the following trinomials.

| $1 . x^{2}-6 x+9$ | $2 \cdot x^{2}+10 x+25$ | $3 \cdot x^{2}-16 x+64$ |
| :--- | :--- | :--- |
|  |  |  |

(a) How does the constant term in the binomial relate to the b term in the trinomial?
(b) How does the constant term in the binomial relate to the c term in the trinomial?

Problems 1-3 are called Perfect Square Trinomials. These trinomials are called perfect square trinomials because when they are in their factored form, they are a binomial squared. An example would be $x^{2}+12 x+36$. Its factored form is $(x+6)^{2}$, which is a binomial squared.

But what if you were not given the c term of a trinomial? Let's see if you can find the missing c term!

Directions: Complete the square for the following expressions. Then factor your expression.
a. $x^{2}+4 x+$ $\qquad$ $=1$
$)^{2}$
b. $x^{2}+8 x+$ $\qquad$ $=1$
$)^{2}$
c. $x^{2}+6 x+$ $\qquad$ $=1$ $)^{2}$ d. $x^{2}+14 x+$ $\qquad$ $=1$
$1^{2}$
e. $x^{2}-2 x+$ $\qquad$ $=1$
$)^{2}$ f. $x^{2}-18 x+$ $\qquad$ $=1$ $1^{2}$
g. $x^{2}-12 x+$ $\qquad$ $=1$
$1^{2}$
h. $x^{2}-20 x+$ $\qquad$ $=1$
$)^{2} \quad$ i. $x^{2}+5 x+$ $\qquad$ $=1$
j. $x^{2}-3 x+$ $\qquad$ $=1$
$)^{2}$
k. $x^{2}-7 x+$ $\qquad$ $=(\quad)^{2}$

1. $x^{2}+9 x+$ $\qquad$ $=1$

## Solving equations via "COMPLETING THE SQUARE":

## The Equation:

STEP 1: move constant term to the other side)
STEP 2: make the left hand side a perfect square trinomial by adding $\left(\frac{b}{2}\right)^{2}$ to both sides

STEP 3: factor the left side, simplify the right side

$$
\begin{aligned}
& x^{2}+6 x+2=0 \\
& x^{2}+6 x+\overline{9}=-2 \\
& x^{2}+6 x+\boxed{9}=-2+9
\end{aligned}
$$

$$
(x+3)^{2}=7 \text { (You've completed the square - time to solve!) }
$$

$$
\sqrt{(x+3)^{2}}=\sqrt{7}
$$

STEP 4: solve by finding square roots

$$
x+3=\sqrt{7} \text { and } x+3=-\sqrt{7}
$$

$$
x=-3+\sqrt{7} \text { and } x=-3-\sqrt{7}
$$

## WE SHOULD ONLY USE THE COMPLETING THE SQUARE METHOD IF:



Group Practice: Solve for $x$.

1. $x^{2}-6 x-72=0$
2. $x^{2}+80=18 x$
$X=$ $\qquad$ $X=$ $\qquad$
3. $x^{2}-14 x-59=-20$

$$
X=
$$

$\qquad$
5. $x^{2}+6 x-21=-6$
$X=$ $\qquad$
4. $2 x^{2}-36 x+10=0$
$X=$ $\qquad$
6. $x^{2}+12 x=-18$
$X=$ $\qquad$

## Day 7 - Finding the Vertex by Completing the Square

Think About It: In Unit 8, you learned to find the vertex of an equation in standard form by finding the $x$-value of the vertex using $x=-b / 2 a$. Today, you are going to learn how to use completing the square to find the vertex.

Take a look at the graph and conversion to standard form. How would you go from $g(x)=x^{2}+8 x+11$ to $g(x)=(x+4)^{2}-5$ ?

$$
g(x)=(x+4)^{2}-5
$$



$$
\begin{gathered}
g(x)=(x+4)^{2}-5 \\
g(x)=(x+4)(x+4)-5 \\
g(x)=x^{2}+8 x+16-5 \\
g(x)=x^{2}+8 x+11
\end{gathered}
$$

Vertex: $(-4,-5)$

## Finding the Vertex by Completing the Square

To finding the vertex from standard form, we are only going to focus on the right side of the equation. Take a look at the following example from above, but this time, we are going from standard to vertex.

| Steps | Reasoning/Justification |
| :---: | :--- |
| $y=x^{2}+8 x+11$ | Original Equation |
| $x^{2}+8 x+\ldots=-11+$ | Move the constant term to the right side |
| $x^{2}+8 x+16=-11+16$ | Determine the missing "c" term |
| $(x+4)^{2}=5$ | Simplify the right side and determine the binomial squared on the <br> left side. |
| $y=(x+4)^{2}-5$ | Move the term on the right back to the other side and set the <br> equation equal to $y$. |
| Vertex: $(-4,-5)$ | Name your vertex. |

## Practice Finding the Vertex by Completing the Square

Find the vertex of the quadratic functions by completing the square.
a. $f(x)=x^{2}+6 x+11$
b. $y=x^{2}-10 x+2$
c. $g(x)=x^{2}+4 x$
d. $y=x^{2}-5 x+4$
e. $y=2 x^{2}-12 x+16$
f. $h(x)=-2 x^{2}+8 x-4$

## Day 8/9: Solving by Quadratic Formula

## Exploring the Nature of Roots

In this task you will investigate the number of real solutions to a quadratic equation.

1. $f(x)=x^{2}-4 x+3$
a.) How many x-intercepts does the function have?
b.) Label and state the $x$-intercept(s), if any.
c.) Solve the quadratic function by factoring, if possible.

2. $f(x)=x^{2}+10 x+25$
a.) How many x-intercepts does the function have?
b.) Label and state the $x$-intercept(s), if any.
c.) Solve the quadratic function by factoring, if possible.
3. $f(x)=x^{2}+x+1$
a.) How many x-intercepts does the function have?
b.) Label and state the x-intercept(s), if any.
c.) Solve the quadratic function by factoring, if possible.


## The Discriminant

Instead of observing a quadratic function's graph and/or solving it by factoring, there is an alternative way to determine the number of real solutions called the discriminant.

| Given a quadratic function in standard form: |
| :---: |
| $a x^{2}+b x+c=0$, where $a \neq 0$, |

The discriminant is found by using: $\boldsymbol{b}^{2}-4 a c$
This value is used to determine the number of real
solutions/zeros/roots/x-intercepts that exist for a quadratic
equation.

Interpretation of the Discriminant ( $b^{2}-4 a c$ )

- If $b^{2}-4 a c$ is positive:
- If $b^{2}-4 a c$ is zero:
- If $b^{2}-4 a c$ is negative:

Practice: Find the discriminant for the previous three functions:
a.) $f(x)=x^{2}-4 x+3$
$a=$ $\qquad$ $b=$ $\qquad$ c = $\qquad$
b.) $f(x)=x^{2}+10 x+25$
$a=$ $\qquad$ $b=$ $\qquad$ $\mathrm{c}=$ $\qquad$
c.) $f(x)=x^{2}+x+1$
$a=$ $\qquad$ $b=$ $\qquad$ $c=$ $\qquad$ Discriminant: $\qquad$ \# of real roots: $\qquad$
Practice: Determine whether the discriminant would be greater than, less than, or equal to zero.




## Quadratic Conundrum

Consider the quadratic equation $x^{2}+3 x-3=0$.
a. How many zeros does this function have?
b. Calculate the discriminant: $\qquad$
c. If possible, factor the quadratic equation.

d. How many solutions does the discriminant of this function imply it would have? Were you able to find these solutions by factoring?

## The Quadratic Formula

We have learned three methods for solving quadratics: factoring, taking the square root, and completing the square. Factoring quadratics only works when the equations are factorable. Taking the square root only works when the equations are not in standard form. Completing the square only works when $a$ is 1 and $b$ is even.

What method do you use when your equations are not factorable, but are in standard form, and a may not be 1 and b may not be even?

The Quadratic Formula
for equations in standard form: $y=a x^{2}+b x+c$

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

x represents the zeros and $\mathrm{b}^{2}-4 \mathrm{ac}$ is the discriminant

## Practice with the Quadratic Formula

For the quadratic equations below, use the quadratic formula to find the solutions. Write your answer in simplest radical form.

1) $4 x^{2}-13 x+3=0 a=$ $\qquad$ $b=$ $\qquad$ $c=$ $\qquad$
2) $9 x^{2}+6 x+1=0 \quad a=$ $\qquad$ $b=$ $\qquad$ $c=$ $\qquad$

Discriminant: $\qquad$ Discriminant: $\qquad$
Solutions: $\qquad$
Approx: $\qquad$
3) $7 x^{2}+8 x+3=0 \quad a=$ $\qquad$ $b=$ $\qquad$ $c=$ $\qquad$ 4) $-3 x^{2}+2 x=-8 \quad a=$ $\qquad$ $b=$ $\qquad$ $\mathrm{c}=$ $\qquad$

Discriminant: $\qquad$
$X=$ $\qquad$
Approx: $\qquad$

Discriminant: $\qquad$
Roots: $\qquad$
Approx: $\qquad$

## Algebra 1

5) $6 x^{2}+3=10 x \quad a=\ldots \quad b=$
6) $\frac{1}{2} x^{2}+6 x+13=0 \quad a=\ldots \quad b=\ldots \quad c=$

Discriminant: $\qquad$ Discriminant: $\qquad$
Solutions: $\qquad$
Approx: $\qquad$
Zeros: $\qquad$
Approx: $\qquad$

## Day 10: Applications of Quadratics

| If you are solving for the vertex: | If you are solving for the zeros: |
| :--- | :--- |
| -Maximum/Minimum (height, cost, etc) | -How long did it take to reach the ground? |
| -Greatest/Least Value | -How long is an object in the air? |
| -Maximize/Minimize | -How wide is an object? |
| -Highest/Lowest | -Finding a specific measurement/dimension |

1. Suppose the flight of a launched bottle rocket can be modeled by the equation $y=-x^{2}+6 x$, where $y$ measures the rocket's height above the ground in meters and $x$ represents the rocket's horizontal distance in meters from the launching spot at $x=0$.
a. How far has the bottle rocket traveled horizontally when it reaches it maximum height? What is the maximum height the bottle rocket reaches?


b. When is the bottle rocket on the ground? How far does the bottle rocket travel in the horizontal direction from launch to landing?

Algebra 1
2. A frog is about to hop from the bank of a creek. The path of the jump can be modeled by the equation $h(x)=-x^{2}+4 x+1$, where $h(x)$ is the frog's height above the water and $x$ is the number of seconds since the frog jumped. A fly is cruising at a height of 5 feet above the water. Is it possible for the frog to catch the fly, given the equation of the frog's jump?

b. When does the frog land back in the water?
c. When will the frog be 3 feet in the air?

## Quadratic Keywords



## Day 11: Determining the Best Method



Determine the best method for solving. Explain why.

1. $6 x^{2}-11 x+3=0$
2. $x^{2}+6 x-45=0$
3. $x^{2}-7 x=8$
4. $8 x^{2}+24 x=0$
5. $2 x^{2}-11 x+5=0$
6. $x^{2}-9 x=-20$

| Non Factorable Methods |  |  |  |
| :---: | :---: | :---: | :---: |
| Completing the Square | Finding Square Roots |  | Quadratic Formula |
| $a x^{2}+b x+c=0$ <br> when $a=1$ and $b$ is an even \# <br> Examples $\begin{aligned} & x^{2}-6 x+11=0 \\ & x^{2}-2 x-20=0 \end{aligned}$ | $a x^{2}-c=0$ <br> Parenthesis in <br> Examples $\begin{aligned} & 2 x^{2}+5=9 \\ & 5(x+3)^{2}-5=20 \\ & x^{2}-36=0 \end{aligned}$ | ation | $a x^{2}+b x+c=0$ <br> Any equation in standard form Large coefficients <br> Examples $\begin{aligned} & 3 x^{2}+9 x-1=0 \\ & 20 x^{2}+36 x-17=0 \end{aligned}$ |
| Factorable Methods |  |  |  |
| A = 1 \& A Not 1 (Factor into 2 Binomials) |  |  | GCF |
| $a x^{2}+b x+c=0$, when $a=1$ $a x^{2} \pm b x \pm c=0$, when $a>1$ $x^{2}-c=0$ <br> Examples $\begin{aligned} & 3 x^{2}-20 x-7=0 \\ & x^{2}-3 x+2=0 \\ & x^{2}+5 x=-6 \\ & x^{2}-25=0 \end{aligned}$ |  | $a x^{2}+b x=0$ <br> Examples $\begin{aligned} & 5 x^{2}+20 x=0 \\ & x^{2}-6 x=8 x \end{aligned}$ |  |

7. $x^{2}-9=0$
8. $x^{2}+4 x+17=0$
9. $2 x^{2}+6 x-37=0$
10. $4(x+4)^{2}=16$
11. $x^{2}-15 x+36=0$
12. $18 x^{2}+100 x=63$
