Learning Goal 10.1: Exponential Functions

After completion of this unit, you will be able to…

Learning Target #1: Graphs and Transformations of Exponential Functions

- Evaluate an exponential function
- Graph an exponential function using a xy chart
- Create an exponential function from a table or graph
- Transform an exponential function by translating, stretching/shrinking, and reflecting
- Identfiy transformations from a function
- Identify domain, range, intercepts, zeros, end behavior, extrema, asymptotes, and intervals of increase/decrease
- Calculate the average rate of change for a specified interval from an equation or graph
- Create exponential models and use them to solve problems

Timeline for Unit 10

Monday	Tuesday	Wednesday	Thursday	Friday
9 *Day 1/2* Intro to Exponentials *&* Graphing and Writing Equatinons of Exponentials	**10** *Day 2/3* Transformations of Exponentials	**11** **Early Release** Finish Day 3 & Practice Day	**12** *Day 4* Characteristics of Exponential Functions	**13** *Day 5* Applications of Exponential Functions
16 Day 6 Application Practice	**17** *Day 7* Review Day	**18** *Day 8* **Unit 10 Test**		

	Monday	Tuesday	Wednesday	Thursday	Friday
AM	Mrs. Jackson 7:45 – 8:15 Room 1210	Mr. Phillips 7:45 – 8:15 Room 1206	Mrs. Jackson & Mr. Webb 7:45 – 8:15 Room 1210 Room 1205	Mr. Watson & Mr. Phillips 7:45 – 8:15 Room 1208 Room 1206	Mr. Watson 7:45 – 8:15 Room 1208
PM	NONE	Mrs. Petersen 3:30 – 4:30 Room 1210	NONE	NONE	NONE

Day 1 – Intro to Exponential Functions

Exploring with Graphs: Graph the following equations:

a. $y = 2x$

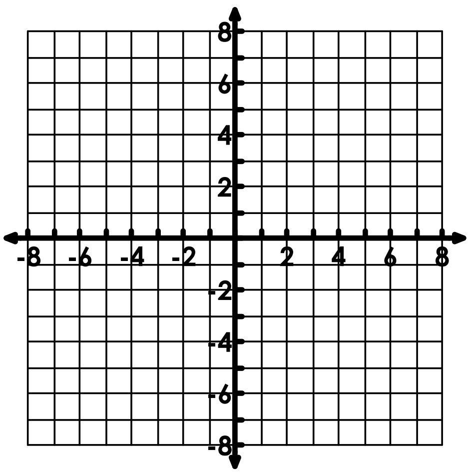
x	-3	-2	-1	0	1	2	3
y							

b. $y = x^2$

x	-3	-2	-1	0	1	2	3
y							

c. $y = 2^x$

x	-3	-2	-1	0	1	2	3
y							



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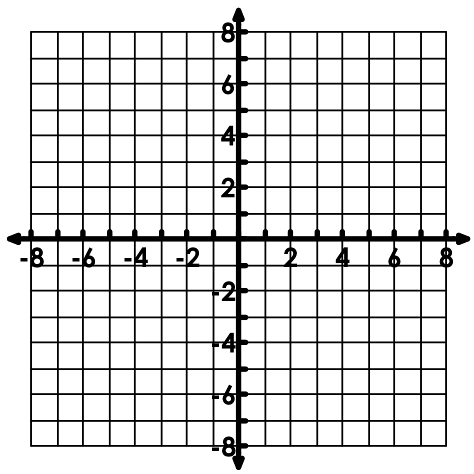

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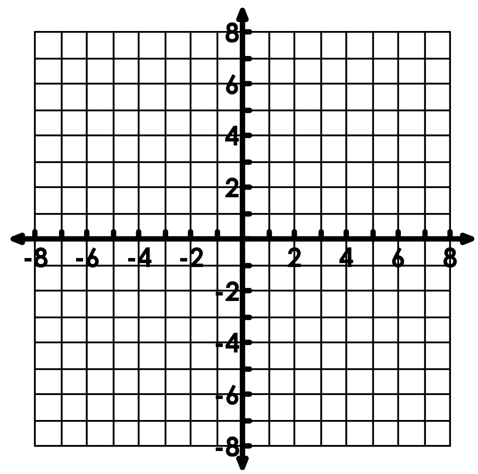

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How is Equation C different from Equations A and B (you have already learned about equations A & B).

Evaluating Exponential Functions

When graphing exponential functions, it is important that you understand how to evaluate an exponential function. Since the variable is in the exponent, you will evaluate the function differently that you did with a linear function. You will still substitute the value of x into the function, but will be taking that value as a power.

Example 1: Evaluate each exponential function.

a. $f(x) = 2(3)^x$ when $x = 5$

b. $y = 8(0.75)^x$ when $x = 3$

c. $f(x) = 4^x$, find $f(2)$.

Exploring with a Scenario:
Which of the options below will make you the most money after 15 days?

a. Earning $100 a day?

x	1	2	3	4	5	6	7	8	9	10
y										

x	11	12	13	14	15	16	17	18	19	20
y										

b. Earning a penny at the end of the first day, earning two pennies at the end of the second day, earning 4 pennies at the end of the third day, earning 8 pennies at the end of the fourth day, and so on?

x	1	2	3	4	5	6	7	8	9	10
y										

x	11	12	13	14	15	16	17	18	19	20
y										

Exponential Functions

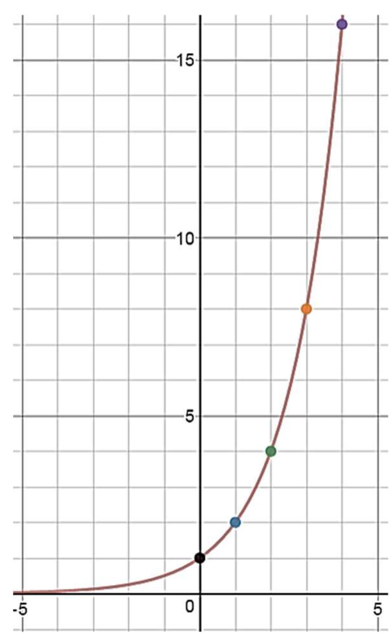
$$y = ab^x$$

1. Variable is in the power (exponent) versus the base

2. Start small and increase quickly or vice versa

3. Asymptotes (heads towards a horizontal line but never touches it)

4. Constant Ratios (multiply by same number every time)

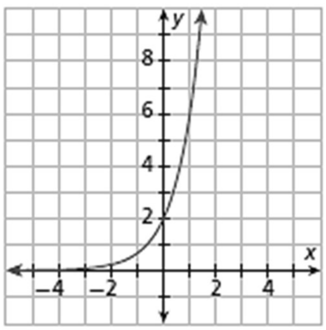


Asymptotes

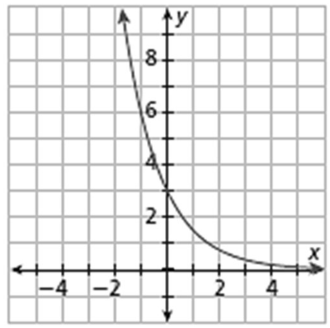
An **asymptote** is a line that an exponential graph gets closer and closer to but never touches or crosses. **The equation for the line of an asymptote for a function in the form of f(x) = ab^x is always y = _______.**

Identify the asymptote of each graph.

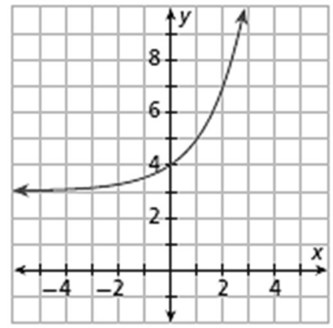
a.



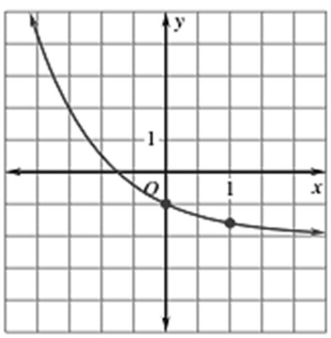
b.



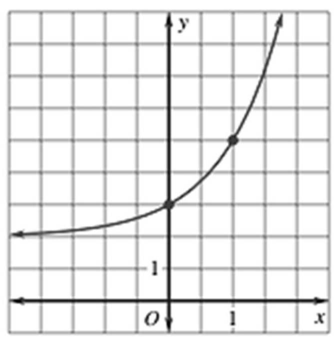
c.



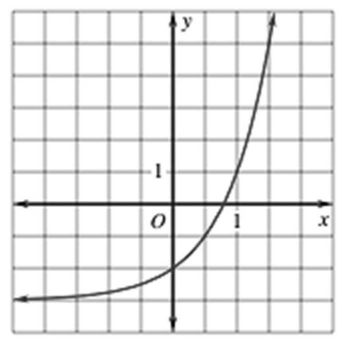
d.



e.



f.



Day 2 - Graphing Exponential Functions

The general form of an exponential function is:

$$y = ab^x$$

Where **a** represents your starting or initial value/population and y-intercept
b represents your growth/decay factor

When you graph exponential functions, you will perform the following steps:

Graphing Exponential Functions Steps

1. Create an x-y chart with 5 values for x (Use table feature to pick 5 values)
2. Substitute those values into the function and record the y or f(x) values.
3. Graph each ordered pair on a graph.

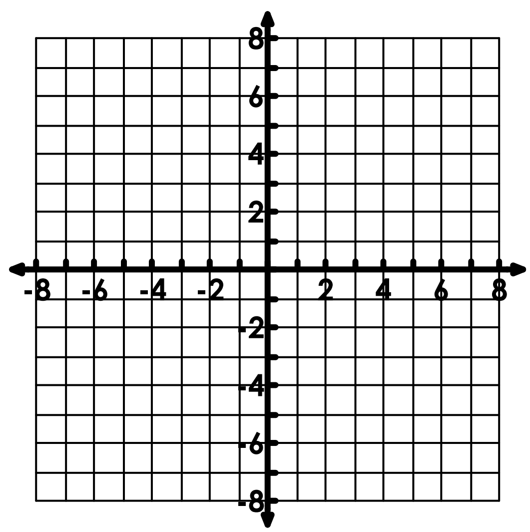
Graph the following:

a. $y = 3(4)^x$

x	y

Y-intercept:

Asymptote:

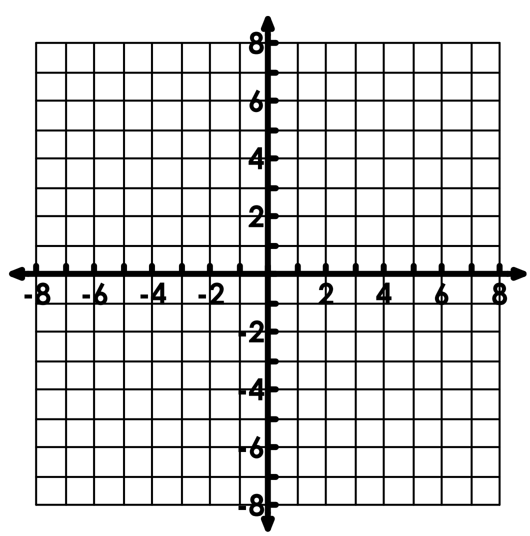


b. $f(x) = 2^x$

x	y

Y-intercept:

Asymptote:

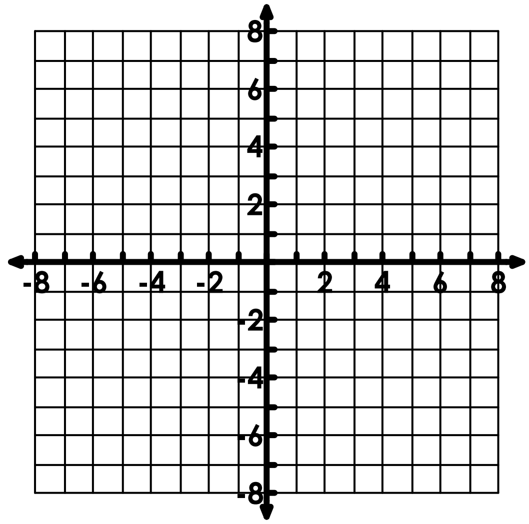


c. $y = 3\left(\frac{1}{2}\right)^x$

x	y

Y-intercept:

Asymptote:

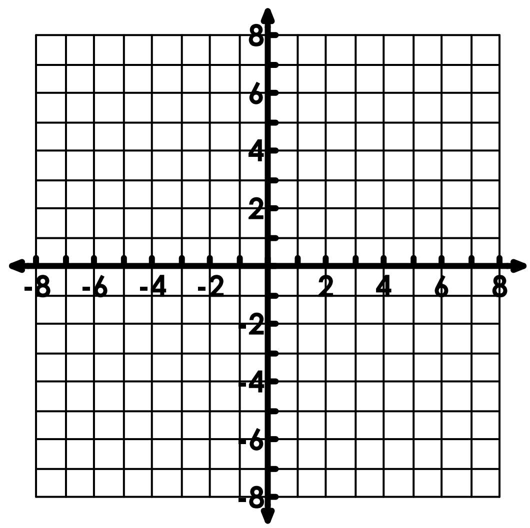


d. $f(x) = 4(.25)^x$

x	y

Y-intercept:

Asymptote:



Think about it…
What did you notice about the y-intercept and the equation?

You have two ways you can find the y-intercept when given an equation: $y = 3(4)^x$

a. ____

b. ____

Summary of Different Types of Exponential Graphs

Equation	'a' values	'b' values	General Shape of Graph
$y = 3(4)^x$ $f(x) = 2^x$			
$y = 3\left(\frac{1}{2}\right)^x$ $f(x) = 4(.25)^x$			

Determine if the following equations represent growth or decay. Then explain why.

a. $y = 4(\frac{3}{4})^x$ b. $y = -2(3)^x$ c. $y = \frac{1}{2}(1.4)^x$ d. $y = 3(\frac{5}{2})^x$

Creating Exponential Functions from Tables and Graphs

The general form of an exponential function is **$y = ab^x$.** You've learned what *a* and *b* represent. The *a* value represents the y-intercept or starting amount and the *b* value represents the constant ratio or growth/decay factor. Therefore, you can think of the general form of an exponential function as the following:

$$y = \text{y-intercept}(\text{constant ratio})^x$$

Using this new idea of thinking about what the equation of an exponential function means, see if you can create an exponential function for each of the tables below:

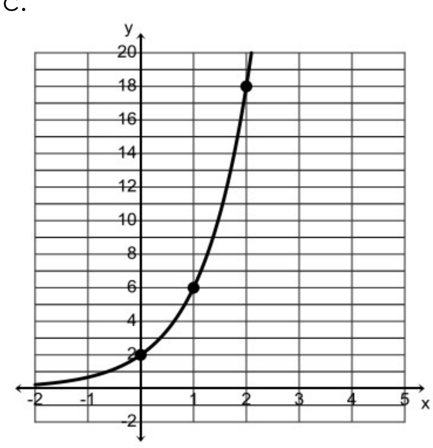
a.

x	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{2}$	2	8	32	128

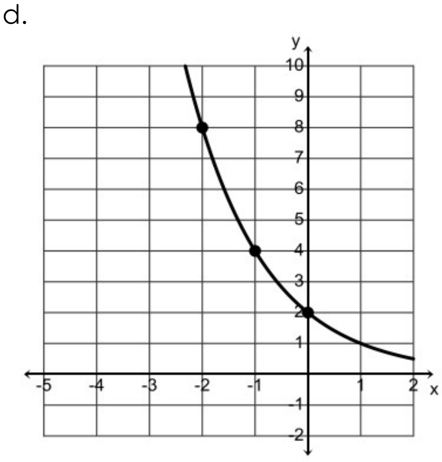
b.

x	-2	-1	0	1	2	3
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

c.



d.



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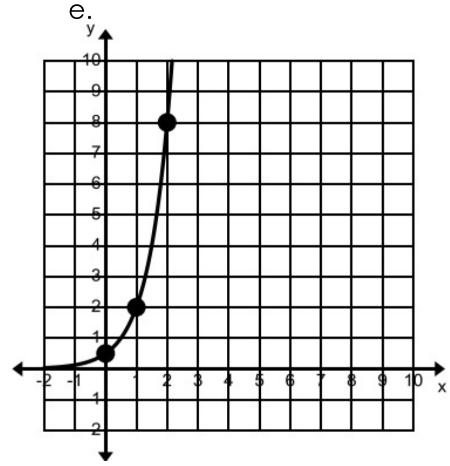


Day 3 – Transformations of Exponential Functions

Transformations of exponential functions is very similar to transformations with quadratic functions. Do you remember what a, h, and k do to the quadratic function?

A: ______________________ H: ______________________ K: ______________________

Summary of Exponential Transformations

The general form of an exponential function is:

$$f(x) = a(b)^{x-h} + k.$$

***When your graph is shifted vertically, the y-intercept becomes a + k.**
***When the graph is shifted vertically, the asymptote becomes y = k.**

If **a** is **negative**, the graph…

If h is **positive**, the graph…

In the equation, I would see…

If h is **negative**, the graph…

In the equation, I would see…

$$y = a(b)^{x-h} + k$$

If **a** is **between 0 and 1**, the graph…

Grows ____________

If **a** is **greater than 1**, the graph…

Grows ____________

If **b** is **greater than 1…**

If **b** is **between 0 & 1…**

If k is **positive**, the graph…

If k is **negative**, the graph…

Asymptote:

Practice Identifying Transformations

Example: Describe the transformations from the parent function to the transformed function:

A. $f(x) = 3^x \rightarrow f(x) = 3^{x+3}$

B. $y = (5)^x \rightarrow y = \frac{1}{2}(5)^x - 4$

C. $y = (0.4)^x \rightarrow y = -3(0.4)^x + 8$

D. $f(x) = 4^x \rightarrow f(x) = 4^{x-6} + 5$

E. $f(x) \rightarrow f(x) + 5$

F. $g(x) \rightarrow g(x + 1)$

G. $f(x) = 3^x \rightarrow f(x) = \frac{3}{4}(3)^{x-2}$

H. $y = 5^x \rightarrow y = -\frac{1}{2}(5)^{x+2}$

I. $y = 0.4^x \rightarrow y = 2(0.4)^x - 6$

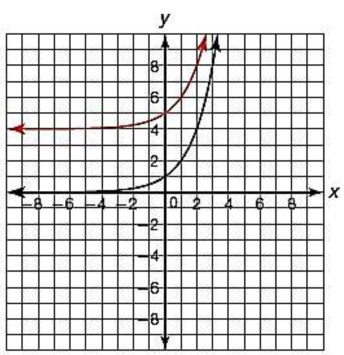
J. $f(x) \rightarrow f(x - 4)$

K. $h(x) \rightarrow 2h(x - 3) - 7$

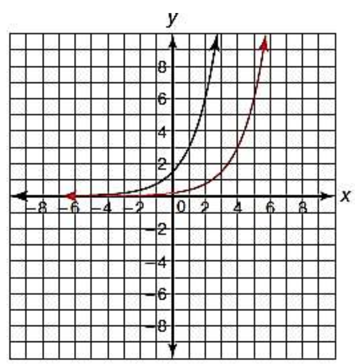
L. $g(x) \rightarrow -g(x + 2) + 1$

Example: Using the graphs of f(x) and g(x), described the transformations from f(x) to g(x):

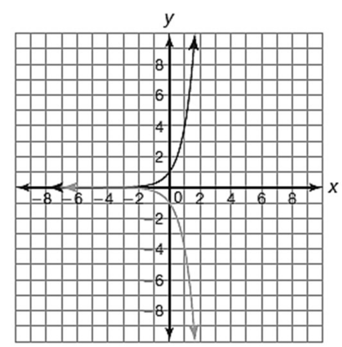
A.



B.



C.



Example: Using the function $g(x) = 5^x$, create a new function h(x) given the following transformations:

A. up 4 units

B. left 2 units

C. down 7 units and right 3 units

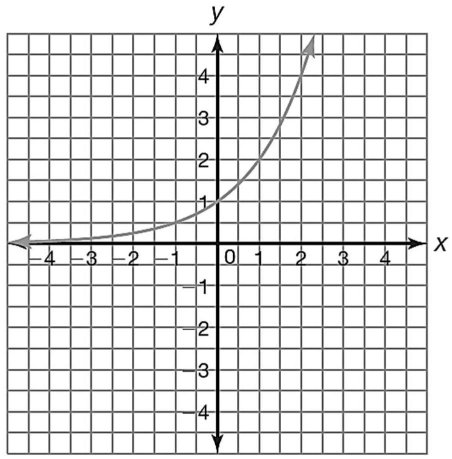
D. stretch by 3

E. reflect over x-axis and left 3

F. Shrink by ½ and reflect over x-axis

Example: Using the graph that is given, y = 2^x, graph a new function with the stated transformations.

a. shifted up two units

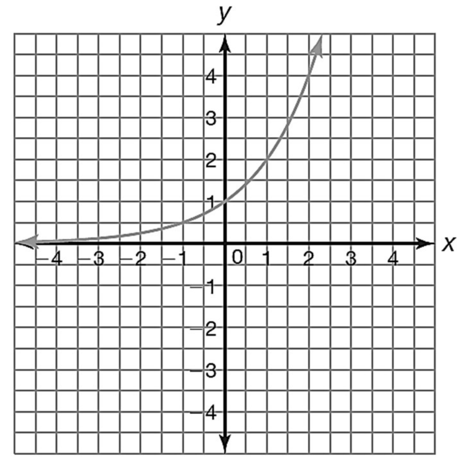


Equation:

Y-intercept:

Asymptote:

b. shifted down 4 units and right 3 units



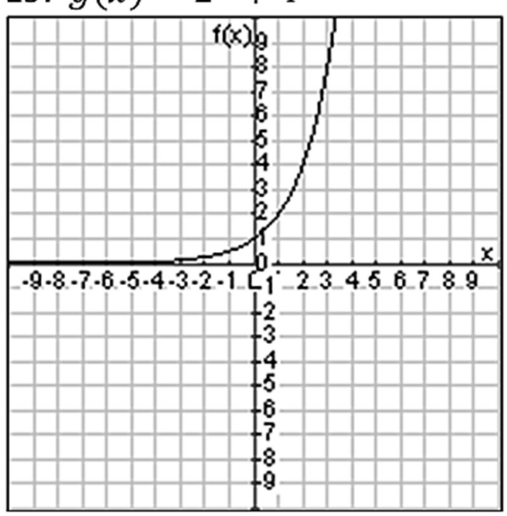
Equation:

Y-intercept:

Asymptote:

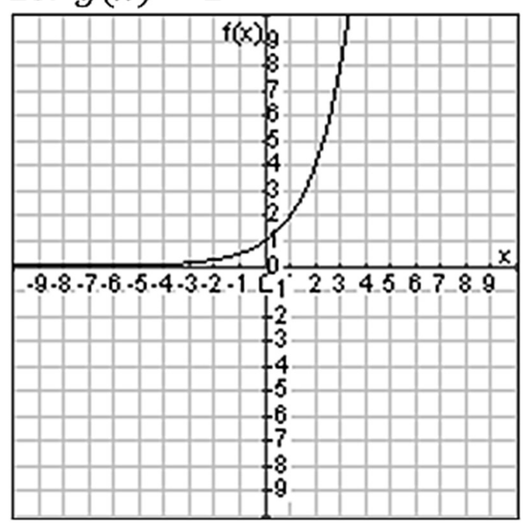
Example: Your parent functions will be f(x) = 2^x. A new function, g(x) is given. Describe the transformations you see in g(x) and then sketch the graph of g(x).

19. $g(x) = 2^x + 4$



Y-intercept: Asymptote:

20. $g(x) = 2^{x+4}$



Y-intercept: Asymptote:

Example: Find the y-intercept and asymptote of the following equations:

A. $f(x) = 3^x \rightarrow f(x) = 3^{x+3}$

y-intercept: asymptote:

B. $y = ½(5)^x \rightarrow y = ½(5)^x – 4$

y-intercept: asymptote:

C. $y = 3(0.4)^x \rightarrow y = 3(0.4)^x + 8$

y-intercept: asymptote:

D. $f(x) = 4^x \rightarrow f(x) = 4^{x-6} + 5$

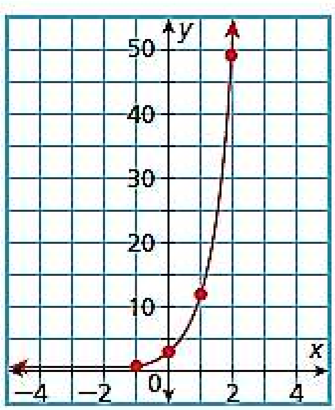
y-intercept: asymptote:

Day 4 – Characteristics of Exponential Functions

As you can hopefully recall, you learned about characteristics of functions in Unit 2 with linear functions and Unit 5 with quadratic functions. We are going to apply the same characteristics, but this time to exponential functions.

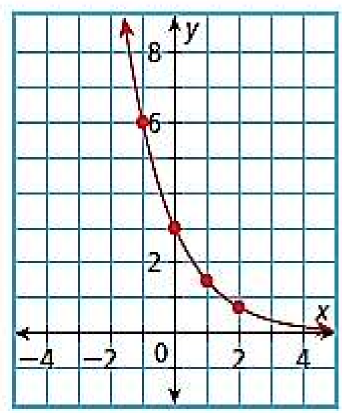
Domain and Range

Domain		
Define: All possible values of x	**Think:** How far left to right does the graph go?	**Write:** Smallest x ≤ x ≤ Biggest x *use < if the circles are open*
Range		
Define: All possible values of y	**Think:** How far down to how far up does the graph go?	**Write:** y < highest y value (opens down) y > lowest y value (opens up)



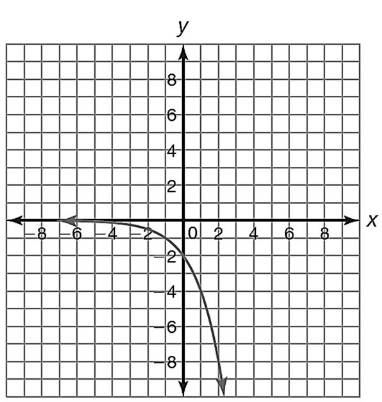
Domain:

Range:



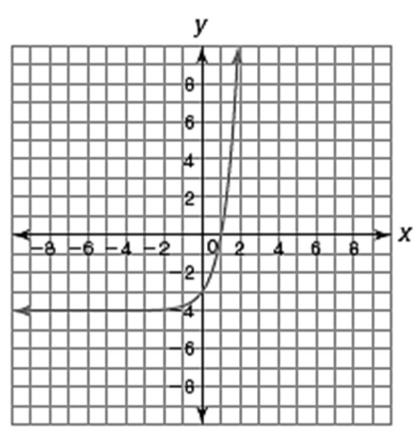
Domain:

Range:



Domain:

Range:

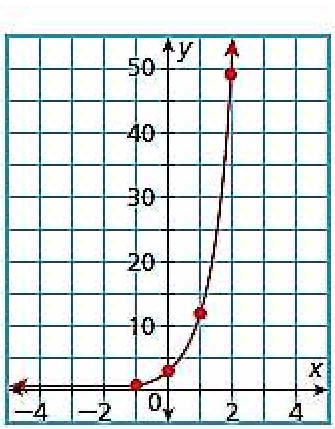


Domain:

Range:

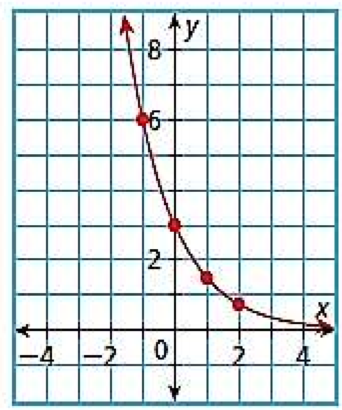
Intercepts and Zeros

Y-Intercept		
Define: Point where the graph crosses the y-axis	**Think:** At what coordinate point does the graph cross the y-axis?	**Write:** (0, b)
X-Intercept		
Define: Point where the graph crosses the x-axis	**Think:** At what coordinate point does the graph cross the x-axis?	**Write:** (a, 0)
Zero		
Define: Where the function (y-value) equals 0	**Think:** At what x-value does the graph cross the x-axis?	**Write:** x = ____



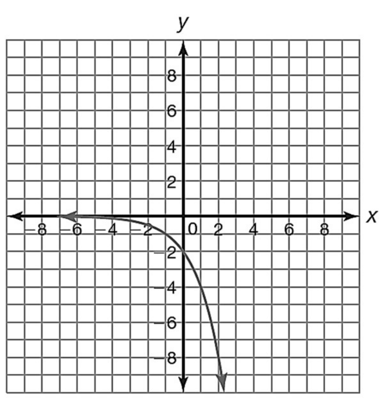
X-intercept: Zero:

Y-intercept:



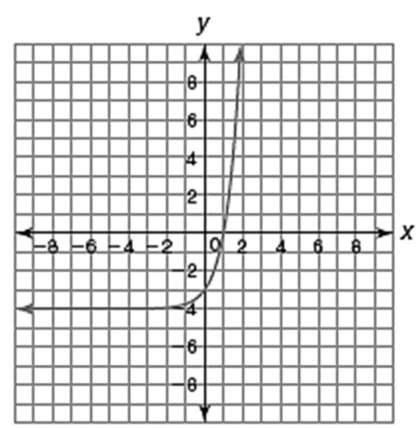
X-intercept: Zero:

Y-intercept:



X-intercept: Zero:

Y-intercept:

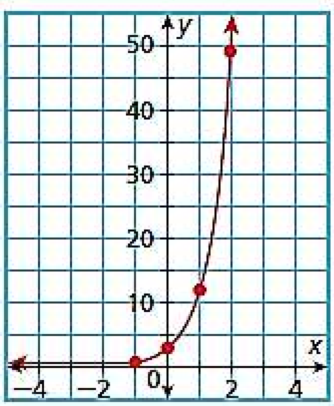


X-intercept: Zero:

Y-intercept:

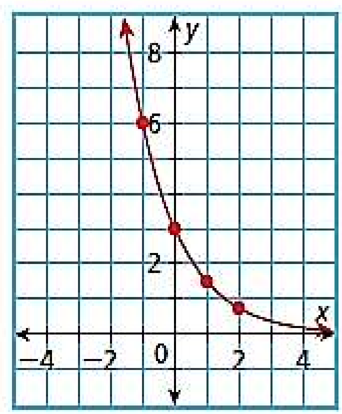
Extremas and Asymptotes

Maximum		
Define: Highest point of a function.	**Think:** What is my highest point on my graph?	**Write:** y =
Minimum		
Define: Lowest point of a function.	**Think:** What is the lowest point on my graph?	**Write:** y =
Asymptotes		
Define: A line that the graph get closer and closer to, but never touches or crosses.	**Think:** What values does my graph begin to flat line towards?	**Write:** y =



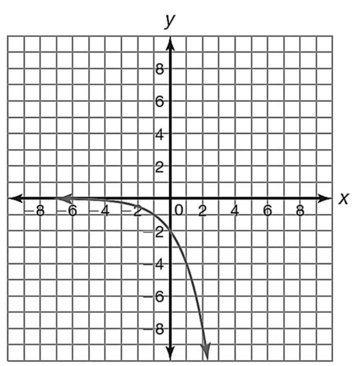
Maximum: Minimum:

Asymptote:



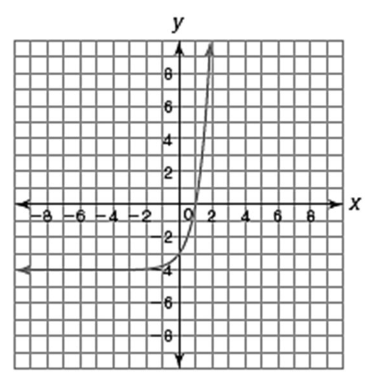
Maximum: Minimum:

Asymptote:



Maximum: Minimum:

Asymptote:



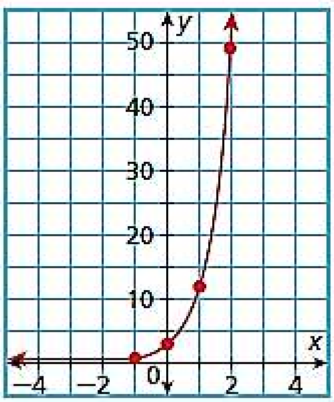
Maximum: Minimum:

Asymptote:

Intervals of Increase and Decrease

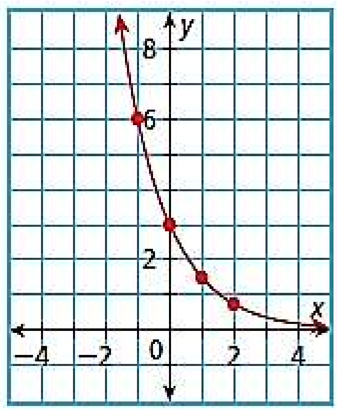
Interval of Increase		
Define: The part of the graph that is rising as you read left to right.	**Think:** From left to right, is my graph going up?	**Write:** An inequality using the x-value of the vertex

Interval of Decrease		
Define: The part of the graph that is falling as you read from left to right.	**Think:** From left to right, is my graph going down?	**Write:** An inequality using the x-value of the vertex



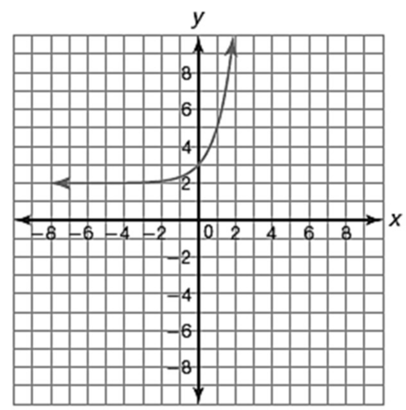
Interval of Increase:

Interval of Decrease:



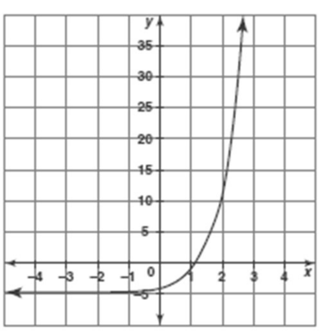
Interval of Increase:

Interval of Decrease:



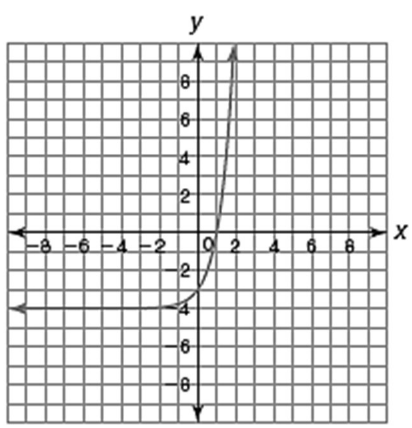
Interval of Increase:

Interval of Decrease:



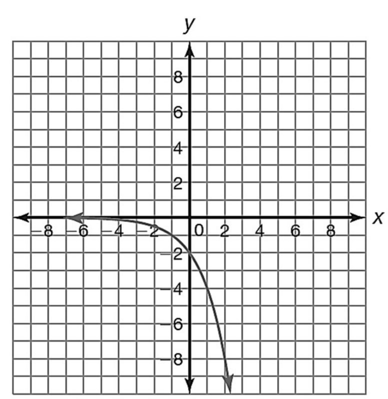
Interval of Increase:

Interval of Decrease:



Interval of Increase:

Interval of Decrease:



Interval of Increase:

Interval of Decrease:

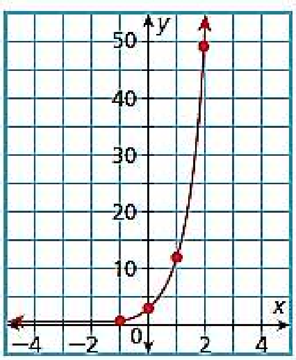
End Behavior

End Behavior

Define:

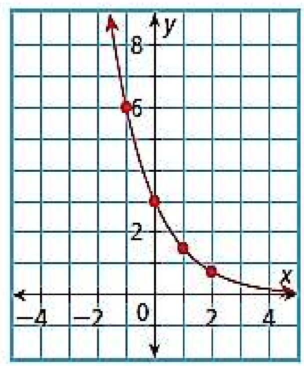
Behavior of the ends of the function (what happens to the y-values or f(x)) as x approaches positive or negative infinity. The arrows indicate the function goes on forever so we want to know where those ends go.

Think:	**Write:**
As x goes to the left (negative infinity), what direction does the left arrow go?	As x → -∞, f(x) → _____
As x goes to the right (positive infinity), what direction does the right arrow go?	As x → ∞, f(x) → _____



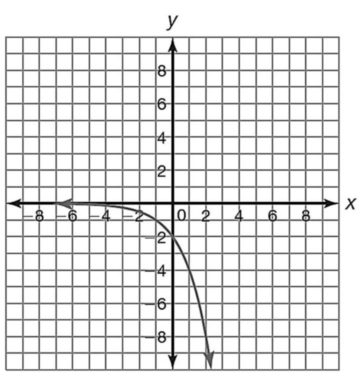
As x approaches -∞, f(x) approaches _____.

As x approaches ∞, f(x) approaches _____.



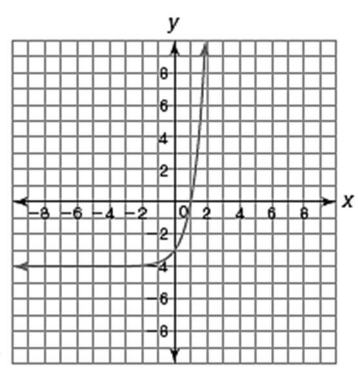
As x approaches -∞, f(x) approaches _____.

As x approaches ∞, f(x) approaches _____.



As x approaches -∞, f(x) approaches _____.

As x approaches ∞, f(x) approaches _____.

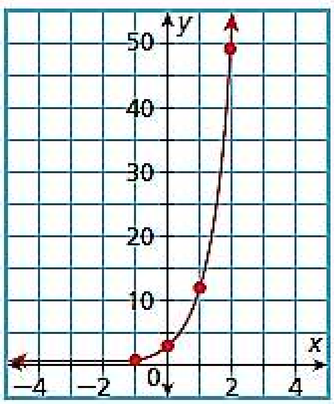


As x approaches -∞, f(x) approaches _____.

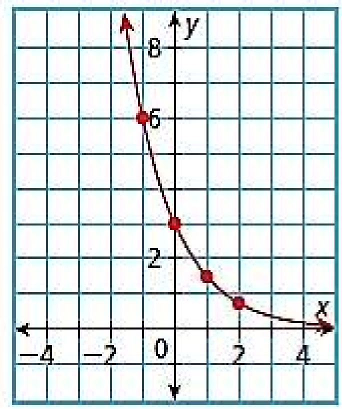
As x approaches ∞, f(x) approaches _____.

Average Rate of Change from a Graph

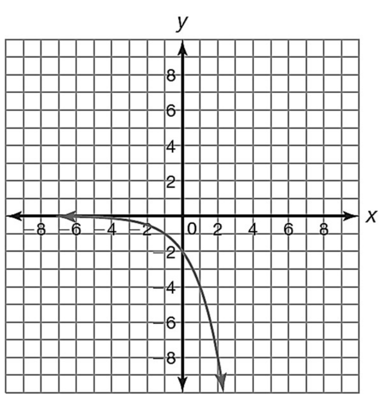
Average Rate of Change: Rate of change or slope for a given interval on a graph. The given interval is written using the inequality notation *a* ≤ x ≤ *b*, where *a* and *b* represent the initial and final x-value of the interval.



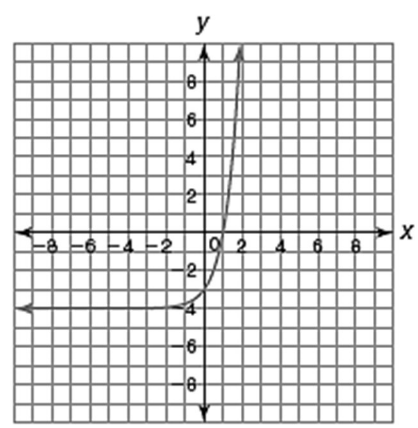
Calculate the average rate of change
for the interval 0 ≤ x ≤ 2



Calculate the average rate of change
for the interval -1 ≤ x ≤ 2



Calculate the average rate of change
for the interval 0 ≤ x ≤ 2



Calculate the average rate of change
for the interval 0 ≤ x ≤ 1

Average Rate of Change from an Equation

If you are given an equation of a function and asked to calculate the average rate of change for that function over a given interval, you will substitute the initial x-value and the final x-value into the function to create two sets of ordered pairs. Then using the ordered pairs, substitute into the slope formula.

a. $y = 3^x$; 1 ≤ x ≤ 3

b. $y = 2(1/2)^x$; -4 ≤ x ≤ 0

Day 5 – Applications of Exponential Functions – Growth/Decay

Review of Percentages: In order to be successful at creating exponential growth and decay functions, it is important you know how to convert a percentage to a decimal. Remember percentages are always out of 100.

Option 1: ________________________________ **Option 2:** ____________________________________

25% = _______ 6.5% = _______ 2% = _______ 10% = _______ 3.05% = _______

Exponential Growth and Decay

As you have already begun to notice, we have been discussing growth and decay quite a bit with exponential functions. You already know how to identify a growth and decay function just from looking at the equation. In case you have forgotten, here are a few practice problems:

A. $y = 8(4)^x$ B. $f(x) = 2(5/7)^x$ C. $h(x) = 0.2(1.4)^x$ D. $y = \frac{3}{4}(0.99)^x$ E. $y = \frac{1}{2}(1.01)^x$

Exponential Growth is where a quantity increases over time where **exponential decay** is where a quantity decreases over time. When we discuss exponential growth and decay, we are going to use a slightly different equation than $y = ab^x$. When you simplify your equation, it will look like $y = ab^x$, but to begin, you will use the following formulas:

Exponential Growth	Exponential Decay
$y = a(1 + r)^t$ where $a>0$ y = final amount a = initial amount r = growth rate (express as decimal) t = time (1 + r) represents the growth factor	**$y = a(1 - r)^t$** where $a>0$ y = final amount a = initial amount r = decay rate (express as decimal) t = time (1 – r) represents the decay factor

Finding Growth and Decay Rates

Example 1: Identify the following equations as growth or decay. Then identify the initial amount, growth/decay factor, and the growth/decay percent.

a. $y = 3.5(1.03)^t$

Growth/Decay: ________

Initial Amount: _________

Growth/Decay Factor: __________

Growth/Decay Percent: _________

b. $f(t) = 10{,}000(0.95)^t$

Growth/Decay: ________

Initial Amount: _________

Growth/Decay Factor: __________

Growth/Decay Percent: _________

c. $g(t) = 400(0.925)^t$

Growth/Decay: ________

Initial Amount: _________

Growth/Decay Factor: __________

Growth/Decay Percent: _________

d. $y = 2{,}500(1.2)^t$

Growth/Decay: ________

Initial Amount: _________

Growth/Decay Factor: __________

Growth/Decay Percent: _________

Growth and Decay Word Problems

Example 2: The original value of a painting is $1400 and the value increases by 9% each year. Write an exponential growth function to model this situation. Then find the value of the painting in 25 years.

Growth or Decay: ____________________________

Starting value (a): ___________

Rate (as a decimal): ________

Function: ____________________________

Example 3: The population of a town is decreasing at a rate of 1% per year. In 2000, there were 1300 people. Write an exponential decay function to model this situation. Then find the population in 2008.

Growth or Decay: ____________________________

Starting value (a): ___________

Rate (as a decimal): ________

Function: ____________________________

Example 4: The cost of tuition at a college is $12,000 and is increasing at a rate of 6% per year. Find the cost of tuition after 4 years.

Growth or Decay: ____________________________

Starting value (a): ___________

Rate (as a decimal): ________

Function: ____________________________

Example 5: The value of a car is $18,000 and is depreciating at a rate of 12% per year. How much will your car be worth after 10 years?

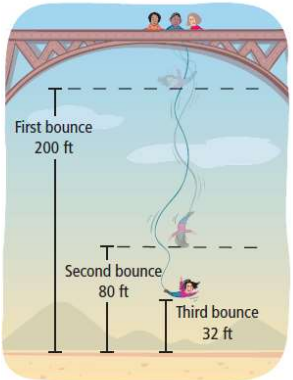
Growth or Decay: ____________________________

Starting value (a): ___________

Rate (as a decimal): ________

Function: ____________________________

Example 6: A bungee jumper jumps from a bridge that is 500 feet high. The diagram shows the bungee jumper's height above the ground at the top of each bounce. What is the bungee jumper's height at the top of the 5^{th} bounce?



Growth or Decay: _______________________

Starting Value: _____________

Rate (as a decimal): _________

Function: _______________________________

Summary of Exponential Word Problems

Creating a Growth Function Given a Percentage Rate

The number of chickens in the farm of Sunny House is currently 2,400. The farm grows at an annual rate of 15%. How many chickens will be there in 7 years?

Growth: $y = a(1 + r)^t$
Increase
Grow
Appreciate
Gains

Creating a Decay Function Given a Percentage Rate

A limousine costs $75,000 new but depreciates at a rate of 23% per year. What is the value of the limousine after five years?

Decay: $y = a(1 - r)^t$
Decreases
Decays
Depreciates
Loses

Creating an Exponential Function without Being Given a Percentage Rate

A 5th grade class is raising meal worms for an experiment. They start with 10 meal worms. The population triples every hour. How many meal worms does the class have after 12 hours?

Special Key Words
Doubles ($b = 2$)
Triples ($b = 3$)
Half ($b = \frac{1}{2}$)
These values replace $(1 \pm r)$

Creating an Exponential Function Given a Pattern

A population of bees is decreasing. The population in a particular region this year is 1250. After year 1, it is estimated that the population will be 1000. After 2 years, it is estimated that the population will be 800. What will the population be in 6 years?

Without a Given Rate:
$y = a \cdot b^x$
a: starting amount
b: multiplier (constant ratio)

Determine if pattern is growth or decay

Working Backwards to Find the Time (Use Table)

The population of a small town has established a growth rate of 3% per year. If the current population is 2000, and the growth rate remains steady, how many years will it take for the population to first go over 3000?

Working Backwards
-Create your equation
-Input into y =
-Use table to find given y-value and its corresponding x-value (t)

Day 6 – Applications of Exponential Functions – Compound Interest

As you get older, you will come to learn a great deal about investing your money…savings accounts, stock market, mutual funds, bonds, etc. Today, we are going to learn about compound interest, which is a form of saving and earning money by letting it sit in an account over time. **Compound Interest** is interest earned or paid on both the principal and previously earned interest. In middle school, you learned about **simple interest**, which is interest that is only earned on the principal. It's formula is $I = Prt$, where P represents principal, r represents rate, t represents time, and I represents interest.

Compound Interest

$$A = P(1+\frac{r}{n})^{nt}$$

A = balance after *t* years
P = Principal (original amount)
r = interest rate (as a decimal)
n = number of times interest is compounded per year
t = time (in years)

Example 1: Write a compound interest function that models an investment of $1000 at a rate of 3% compounded quarterly. Then find the balance after 5 years.

P = _________

r = _________

n = _________

t = __________

Example 2: Write a compound interest function that models an investment of $18,000 at a rate of 4.5% compounded annually. Then find the balance after 6 years.

P = _________

r = _________

n = _________

t = __________

Example 3: Write a compound interest function that models an investment of $4,000 at a rate of 2.5% compounded monthly. Then find the balance after 10 years.

P = _________

r = _________

n = _________

t = __________