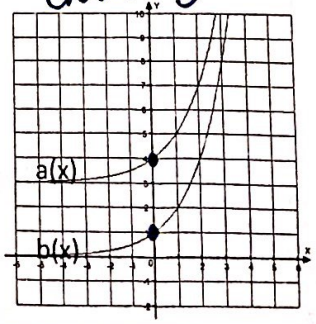
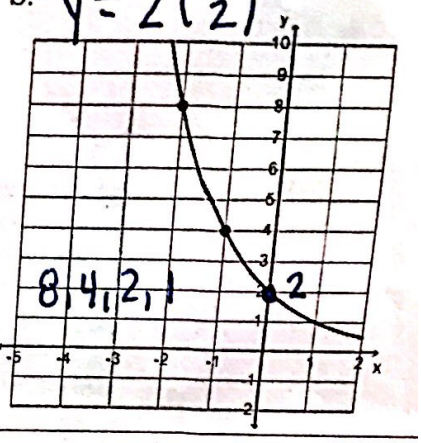
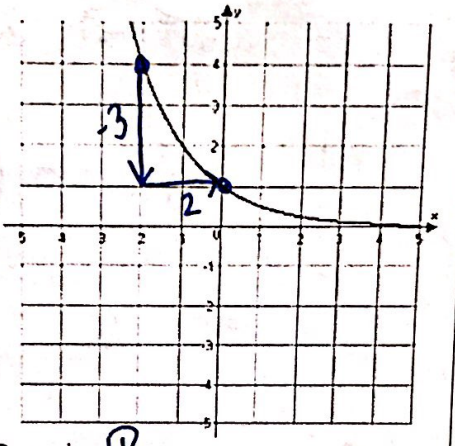
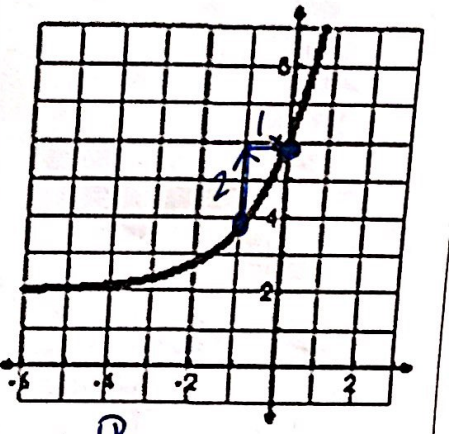
Algebra 1
Unit 10 Review Guide

Name: Key
Date: _____ Block: _____

Exponential Functions Unit Review

Skill	Things to remember	Examples	
1. Determine if representations are exponential. Explain why or why not	Exponential Functions: -Variable in exponent -Constant Ratios -Graph is a curve Linear Functions: -Constant differences -Graph is a line	a. Determine if the points are exponential or linear: a. exponential x: -3, -2, -1, 0, 1 y: 0.16, 0.8, 4, 20, 100 x5 x5 b. (-2, 5) (-1, 4) (0, 3) (1, 2) (2, 1) linear -1 -1	b. Determine if the equations are linear or exponential: a. $y = 3^x - 4$ exponential b. $y = 2^2$ linear c. $y = 6^{2x}$ exponential
2. Determine if a function is exponential growth or decay and explain why.	$0 < b < 1$: Decay $b > 1$: Growth	a. $y = .75\left(\frac{3}{2}\right)^x$ Growth ($3/2 > 1$)	b. $y = \left(\frac{1}{2}\right)^x$ Decay ($0 < 1/2 < 1$)
		c. What is the function growing by? $Y = 3(2)^x$ 2	d. What is constant ratio? $Y = 3(4.5)^x$ 4.5
3. Graph an exponential function.	$y = ab^x$ Create a table with values (5 points is a must)	a. Graph: $f(x) = \left(\frac{1}{2}\right)^x$ x | y -3 | 8 -2 | 4 -1 | 2 0 | 1 1 | 0.5 2 | 0.25	b. Graph: $f(x) = 3 \cdot 2^{x-1} + 1$ x | y -3 | 1.187 -2 | 1.375 -1 | 1.75 0 | 2.5 1 | 4 2 | 7
4. Describe the transformations of an exponential function.	$f(x) = a(b)^{x-h} + k$ **a** stretches or shrinks AND/OR reflects **k** moves the function up and down. **h** moves the function left and right. The new asymptote is the line $y = k$.	a. Given the function $f(x) = 2^x$ write a new equation after a transformation of left 7 and up 3. $f(x) = 2^{x+7} + 3$	b. Given the function $g(x) = 2^x$, write a new equation after a transformation of right 9 and reflect across the x-axis. $g(x) = -2^{x-9}$
		c. Describe the transformation $h(x) = 10^x$ to $k(x) = 4(10)^{x+1} - 5$. Stretch by 4 Left 1 down 5	d. Describe the transformation from a(x) to b(x). down 3 

Create equations from a graph or table	$y = y\text{-int}(\text{constant ratio})^x$	a. $y = \frac{1}{16}(4)^x$	b. $y = 2(\frac{1}{2})^x$ 
6. Determine characteristics of exponential functions.		a.  Domain: $\mathbb{R}$ Range: $y > 0$ x-Intercept: none y-intercept: (0,1) Interval of Increase: none Interval of Decrease: $\mathbb{R}$ Asymptote: $y = 0$ End Behavior: as $x \to -\infty$, $f(x) \to \infty$ as $x \to \infty$, $f(x) \to 0$ ROC from -2 to 0: $-3/2$	b.  Domain: $\mathbb{R}$ Range: $y > 2$ x-Intercept: none y-intercept: (0,6) Interval of Increase: $\mathbb{R}$ Interval of Decrease: none Asymptote: $y = 2$ End Behavior: as $x \to -\infty$, $f(x) \to 2$ as $x \to \infty$, $f(x) \to \infty$ ROC from -1 to 0: 2
7. Determine the y-intercept and asymptote from an equation	You can always substitute 0 in for x to find a y-intercept Asymptote: y = k No 'k' value, the asymptote is y = 0.	a. Determine the y-intercept and asymptote of the function $y = 3(2)^x$. asymptote: $y = 0$ y-int: (0,3)	b. Determine the y-intercept and asymptote of the function $y = 4(\frac{1}{2})^x - 2$. Asymptote: $y = -2$ y-int: (0,2)
8. Average Rate of Change	$m = \frac{y_2 - y_1}{x_2 - x_1}$ Use table to find specified points	a. $f(x) = 2(\frac{1}{5})^x$ for x = -1 and x = 0 (-1,10) (0,2) $m = \frac{2-10}{0-(-1)} = \frac{-8}{1} = \boxed{-8}$	b. $g(x) = \frac{1}{2}(3)^{x+1}$ for [0, 5] (0,1.5) (5,364.5) $m = \frac{364.5 - 1.5}{5-0}$ $= \boxed{\frac{363}{5} \text{ or } 72.6}$

9. Determine the growth/decay factor and percent.

(1 + r) and (1 − r) represent the growth and decay factors

Percent is just the r value

a. $y = 3(1.25)^x$

Determine if the function is growth or decay: Growth

Factor: 1.25

Percent: 25% Change

100% + ? = 125%

b. $y = 2(.84)^x$

Determine if the function is growth decay: Decay

Factor: .84

Percent: 16% Change

100% − ___ = 84%

10. Applications of exponential functions.

$y = a(1 + r)^t$

$y = a(1 - r)^t$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

a. Duke deposits $2000 into a bank account that pays 5% interest compounded monthly. Find the balance in the account after 4 years.

Model: $A = 2000\left(1 + \frac{.05}{12}\right)^{12t}$

$A = 2000\left(1 + \frac{.05}{12}\right)^{12 \cdot 4}$

Solution: $2441.79

b. The value of the Barbie Dream House is $125,000. This house is in a prime location and appreciates (increases in value) at a rate of 7% per year. How much will the Barbie Dream House be worth in 5 years?

Model: $y = 125{,}000(1.07)^t$

$y = 125{,}000(1.07)^5$

Solution: $175,319

c. A certain radioactive element decays at a rate of 21% per month. If the starting amount was 32 ounces, how much will be left after 1 year?

Model: $y = 32(.79)^t$ (t → 12)

$y = 32(.79)^{12}$

Solution: y = 1.89 ounces

d. Michael is offered two jobs – Job A, which offers him a starting salary of $20,000 a year with a 5% raise each year he works there and Job B, which offers him a starting salary of $25,000, but only a 3% raise each year. Michael plans to work to work at the job for 7 years. Which job should he pick and why?

Job A: $y = 20{,}000(1.05)^t$

$y = 20{,}000(1.05)^7$

$y = \$28{,}142$

Job B: $y = 25{,}000(1.03)^t$

$y = 25{,}000(1.03)^7$

$y = \$30{,}747$

Job B pays more after 7 years.

11. Solve an exponential equation.

*Rewrite the bases using exponents.

a. $4^{x+2} = 4^{4x-1}$

$x + 2 = 4x - 1$

$-x + 1 \quad -x + 1$

$3 = 3x$

$\frac{3}{3} \quad \frac{3}{3}$

$1 = x$

b. $5^{3x-3} = 125$

$5^{3x-3} = 5^3$

$3x - 3 = 3$

$+3 \quad +3$

$\frac{3x}{3} = \frac{6}{3}$

$x = 2$

c. $9^{3x+16} = 81^{x+5}$

$9^{3x+16} = 9^{2(x+5)}$

$3x + 16 = 2x + 10$

$-2x - 16 \quad -2x - 16$

$x = -6$

d. $4^{2x} = 64$

$4^{2x} = 4^3$

$\frac{2x}{2} = \frac{3}{2}$

$x = 3/2$

12. Geometric Sequences

Explicit: $a_n = a_1 \cdot r^{n-1}$

Recursive: $a_1 =$ ____
$a_n = r(a_{n-1})$

You must always know your first term and the common ratio to write an explicit formula!

a. Create an explicit and recursive formula for the following:

2, 6, 18, 54, ...

$a_1 = 2 \quad r = 3$

Explicit: $a_n = 2(3)^{n-1}$

Recursive: $a_1 = 2$
$a_n = 3(a_{n-1})$

b. Create an explicit and recursive formula for the following:

81, 27, 9, 3, ...

$a_1 = 81 \quad r = 1/3$

Explicit: $a_n = 81(1/3)^{n-1}$

Recursive: $a_1 = 81$
$a_n = \frac{1}{3}(a_{n-1})$

c. Determine the 12th term in the sequence: 5, 15, 45, ...

$a_1 = 5 \quad r = 3$

$a_n = 5(3)^{n-1}$

$a_{12} = 5(3)^{12-1}$

$\boxed{a_{12} = 885{,}735}$

d. Determine the 10th term in the sequence: 0.1, 0.5, 2.5, ...

$a_1 = 0.1 \quad r = 5$

$a_n = 0.1(5)^{x-1}$

$a_{10} = 0.1(5)^{10-1}$

$\boxed{a_{10} = 195{,}312.5}$

e. Determine the first five terms of the sequence: $a_n = -2 \cdot 3^{n-1}$

$-2, -6, -18, -54, -162$

f. Determine the first five terms of the sequence: $a_1 = 6$, $a_n = \frac{1}{2}(a_{n-1})$

$6, 3, 1.5, .75, .375$

g. Write the explicit formula given the following:

$a_4 = 192$ and $a_5 = 768$

1	2	3	4	5
3	12	48	192	768

(×4 between each pair)

$\boxed{a_n = 3(4)^{x-1}}$

h. Write the explicit formula given the following:

$a_2 = -6$ and $a_3 = -18$

1	2	3
-2	-6	-18

(×3 between each pair)

$\boxed{a_n = -2(3)^{x-1}}$