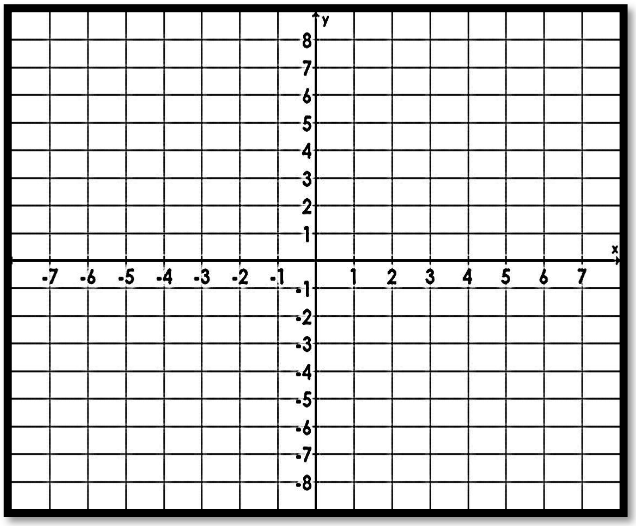
Day 1: Quadratic Transformations (H & K values)

The **parent function** of a function is the simplest form of a function. The parent function for a quadratic function is $y = x^2$ or $f(x) = x^2$. Graph the parent function below.

x	x^2
-3	
-2	
-1	
0	
1	
2	
3	



As you can see, the graph of a quadratic function is very different than the graph of a linear function.

The U-shaped graph of a quadratic function is called a ______________________________.

The highest or lowest point on a parabola is called the ______________________________.

One other characteristic of a quadratic equation is that one of the terms is always ____________________.

There are several different forms a quadratic function can be written in, but the one we are going to work with for today is called **vertex form**. In the following explorations below, you are going to learn the effect of a, h, and k values have on the parent graph.

Vertex Form

$$f(x) = a(x-h)^2 + k$$

Vertex: ____________

Go to: www.student.desmos.com

Enter the code: 2GR6FZ

Discovering Quadratic Transformations with *Desmos*

<u>Slide 1 ~ The K Value ~ $y = x^2 + k$</u>

a. What does the k value do the blue graph? ______________________________

b. What does a positive k value do to the blue graph? ______________________________

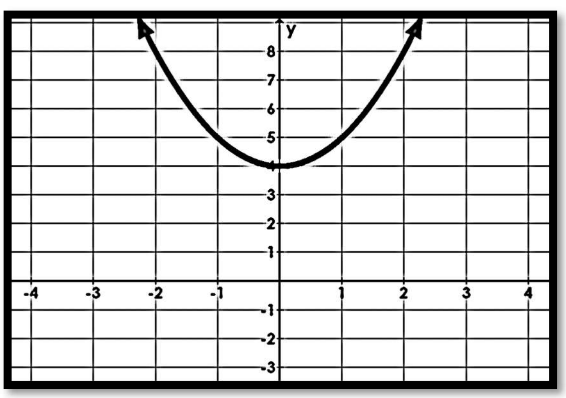
c. What does a negative k value do the blue graph? ______________________________

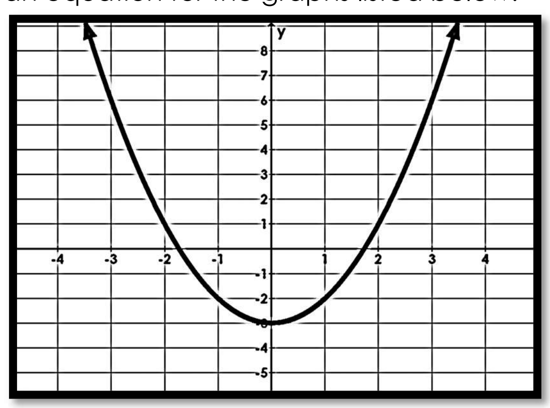
d. Which coordinate of the vertex changes when there is a k value present? ______________________________

e. Name the transformations that would occur for the following equations (you may use the regular Desmos calculator for help). Then name the vertex.

Equation	Transformations	Vertex
1. $y = x^2 + 5$		
2. $y = x^2 - 3$		
3. $y = x^2 + 7$		
4. $y = x^2 - 4$		

f. Describe the transformations and name the vertex. Create an equation for the graphs listed below.





g. Given the transformations listed below, create an equation that would represent the transformations.

1. Shifted up 8 units

2. Shifted up 20 units

3. Shifted down 5 units

Slide 2 ~ The H Value ~ $y = (x - h)^2$

a. What does the h value do the blue graph? ______________________________

b. What does a positive h value do to the blue graph? ______________________________

c. What does a negative h value do the blue graph? ______________________________

d. Which coordinate of the vertex changes when there is an h value present? ______________

Slide 3 ~ The Tricky Part about the H Value

e. Compare the blue graph to the black graph. How did the blue graph move? ______________

f. What should be the h-value for the blue graph? ________

g. However, when you look at the equation for the blue graph, what do you notice?

h. Compare the green graph to the black graph. How did the green move? ______________

i. What should be the h-value for the green graph? ________

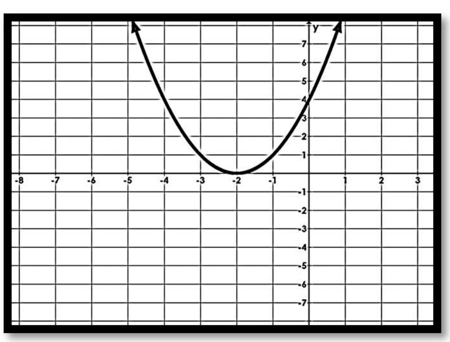
j. However, when you look at the equation for the green graph, what do you notice?

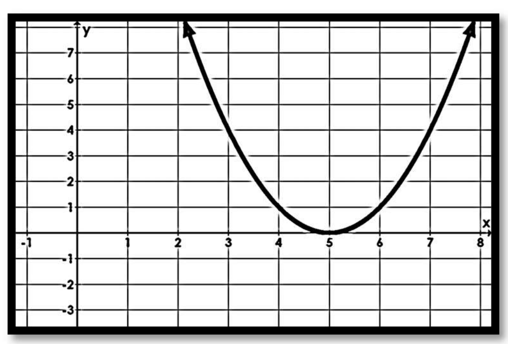
HMMM…..Now read Slide 4!

k. Name the transformations that would occur for the following equations (you may use the regular Desmos calculator for help). Then name the vertex.

Equation	Transformations	Vertex
1. $y = (x - 4)^2$		
2. $y = (x + 6)^2$		
3. $y = (x - 7)^2$		
4. $y = (x + 3)^2$		

l. Describe the transformations and name the vertex. Create an equation for the graphs listed below.





m. Given the transformations listed below, create an equation that would represent the transformations.

1. Shifted right 8 units

2. Shifted left 20 units

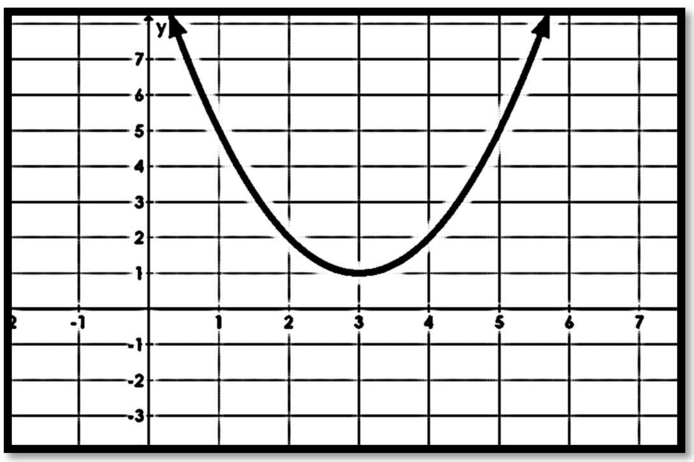
3. Shifted left 5 units

Putting It All Together with H and K

Practice: Identify the transformations and vertex from the equations below.

Equation	Transformations	Vertex
1. $y = (x - 2)^2 + 4$		
2. $y = (x + 3)^2 - 2$		
3. $y = (x - 9)^2 - 5$		
4. $y = (x + 5)^2 + 6$		

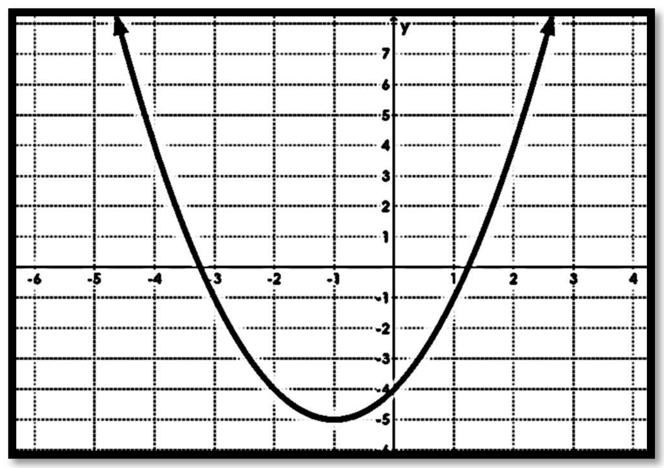
Practice: Describe the transformations and name the vertex. Create an equation for the graphs listed below.



Transformations:

Vertex:

Equation:



Transformations:

Vertex:

Equation:

Practice: Given the transformations listed below, create an equation that would represent the transformations.

1. Shifted up 4 units and left 3 units

2. Shifted right 5 units and down 2 units

3. Shifted left 8 units and down 1 unit

4. Shifted up 5 units and right 9 units

Slide 5 ~ The A Value, part 1 ~ $y = ax^2$

a. What does the a value do the blue graph? ______________________________

b. When a is greater than 1, what does it do to the blue graph? ______________________________

c. When a is between 0 and 1, what does it do to the blue graph? ______________________________

d. If there is only an a value, what will the vertex always be? ____________________

Slide 6 ~ The A Value, part 2 ~ $y = ax^2$

a. What does the a value do the blue graph? ______________________________

b. When a is less than 1, what does it do to the blue graph? ______________________________

Practice: Describe the transformations from the given function to the transformed function.

a. $f(x) = x^2 \rightarrow f(x) = 4x^2$ b. $y = x^2 \rightarrow y = \frac{1}{4}x^2$ c. $f(x) \rightarrow 6f(x)$

d. $f(x) = x^2 \rightarrow f(x) = -x^2$ f. $y = x^2 \rightarrow y = -\frac{1}{2}x^2$ g. $f(x) \rightarrow -4f(x)$

Putting It All Together with A, H, and K

Practice: Given the equations below, name the vertex and describe the transformations:

Equation	Transformations	Vertex
1. $y = -(x-4)^2 + 7$		
2. $y = -2(x+2)^2 + 5$		
3. $y = \frac{1}{2}(x-3)^2 - 8$		

Practice: Create an equation to represents the following transformations:

a. Shifted down 4 units, right 1 unit, and reflected across the x-axis

b. Shifted up 6 units, reflected across the x-axis, and stretch by a factor of 3

c. Shifted up 2 units, left 4 units, reflected across the x-axis, and shrunk by a factor of ¾.

Day 2 - Characteristics of Quadratics

One key component to fully understanding quadratic functions is to be able to describe characteristics of the graph and its equation.

Domain and Range

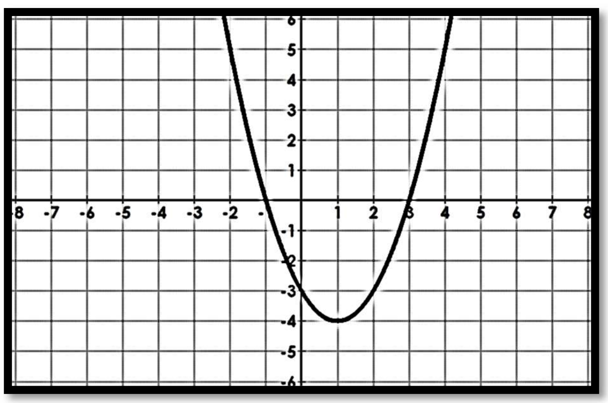
Domain

Define:	Think:	Write:
All possible values of x	How far left to right does the graph go?	Smallest x ≤ x ≤ Biggest x *use < if the circles are open*

Range

Define:	Think:	Write:
All possible values of y	How far down to how far up does the graph go?	y ≤ highest y value (opens down) y ≥ lowest y value (opens up)

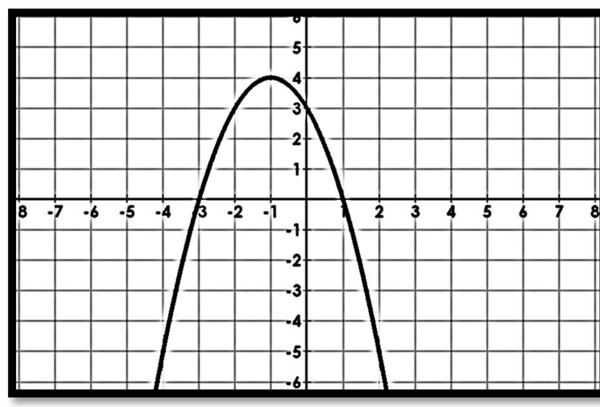
Graph 1



Domain:

Range:

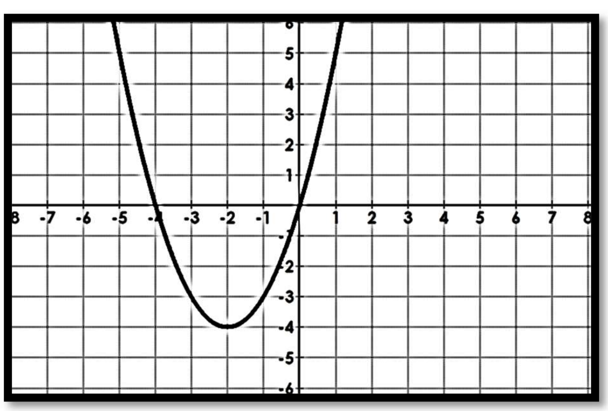
Graph 2



Domain:

Range:

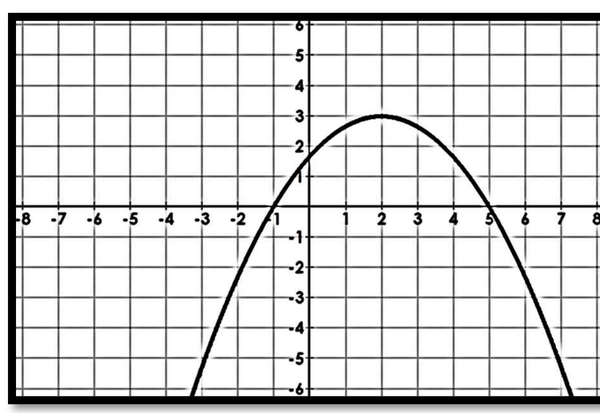
Graph 3



Domain:

Range:

Graph 4



Domain:

Range:

Zeros and Intercepts

Y-Intercept

Define:	Think:	Write:
Point where the graph crosses the y-axis	At what coordinate point does the graph cross the y-axis?	(0, b)

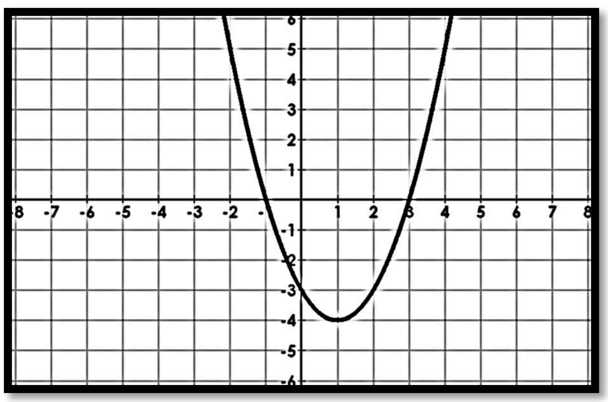
X-Intercept

Define:	Think:	Write:
Point where the graph crosses the x-axis	At what coordinate point does the graph cross the x-axis?	(a, 0)

Zero

Define:	Think:	Write:
Where the function (y-value) equals 0	At what x-value does the graph cross the x-axis?	x = ____

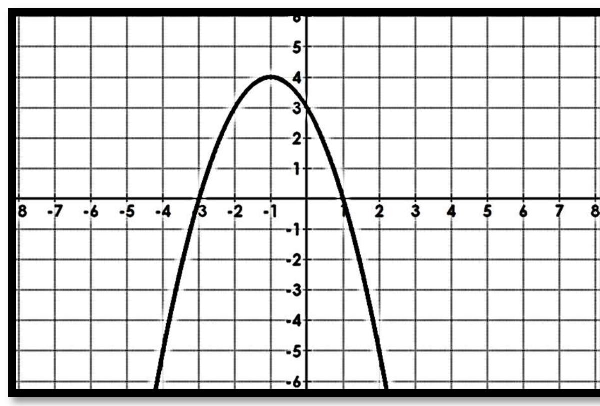
Graph 1



X-intercepts: Y-intercept:

Zeros:

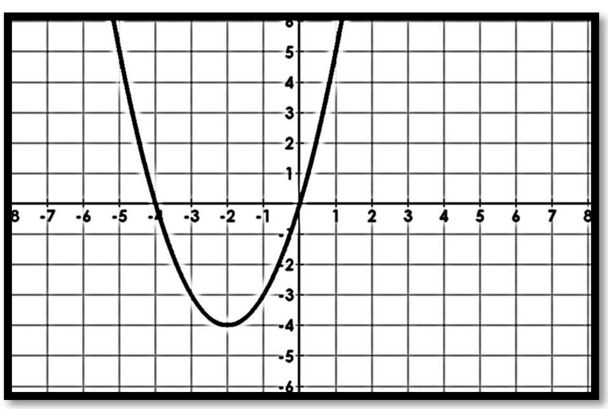
Graph 2



X-intercepts: Y-intercept:

Zeros:

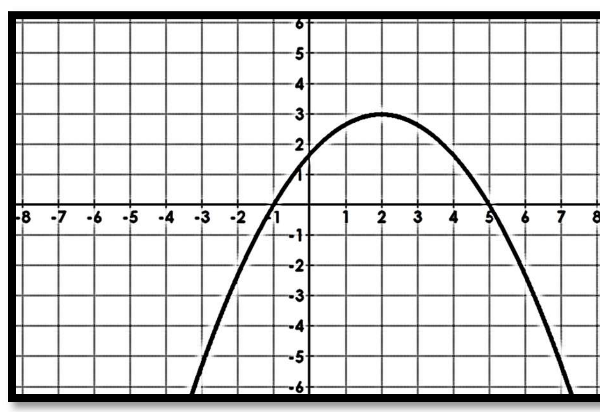
Graph 3



X-intercepts: Y-intercept:

Zeros:

Graph 4



X-intercepts: Y-intercept:

Zeros:

Vertex & Axis of Symmetry

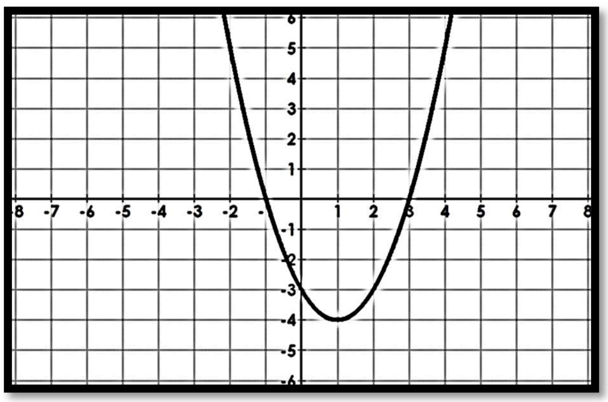
Vertex

Define:	Think:	Write:
Highest or lowest point or peak of a parabola	What is my highest or lowest point on my graph?	Name the point (h, k)

Axis of Symmetry

Define:	Think:	Write:
The vertical line that divides the parabola into mirror images and runs through the vertex	What imaginary, vertical line would make the parabola symmetrical?	x = h (x value of the vertex)

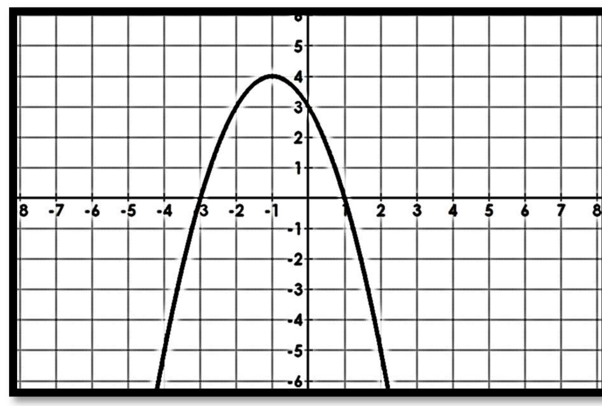
Graph 1



Vertex:

Axis of Symmetry:

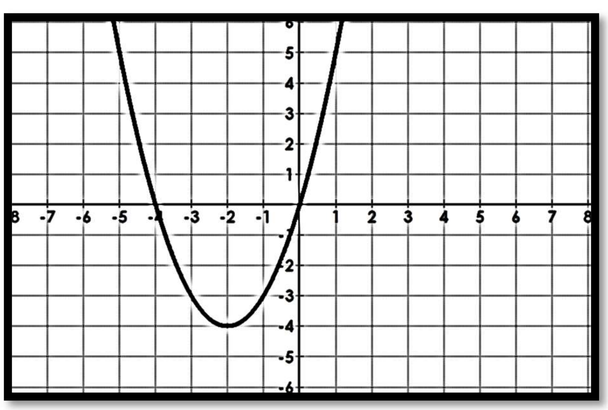
Graph 2



Vertex:

Axis of Symmetry:

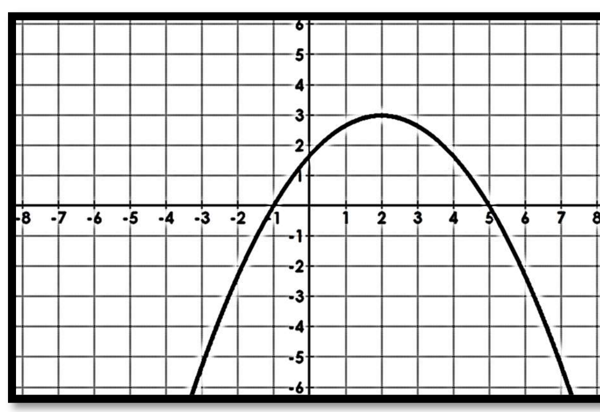
Graph 3



Vertex:

Axis of Symmetry:

Graph 4



Vertex:

Axis of Symmetry:

Extrema

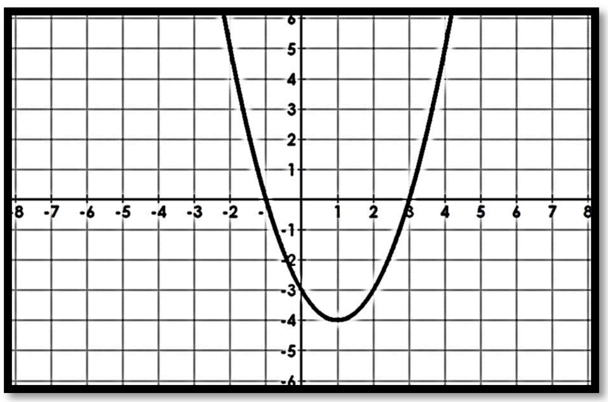
Maximum

Define:	Think:	Write:
Highest point or peak of a function.	What is my highest point on my graph?	y = k (y-value of the vertex)

Minimum

Define:	Think:	Write:
Lowest point or valley of a function.	What is the lowest point on my graph?	y = k (y-value of the vertex)

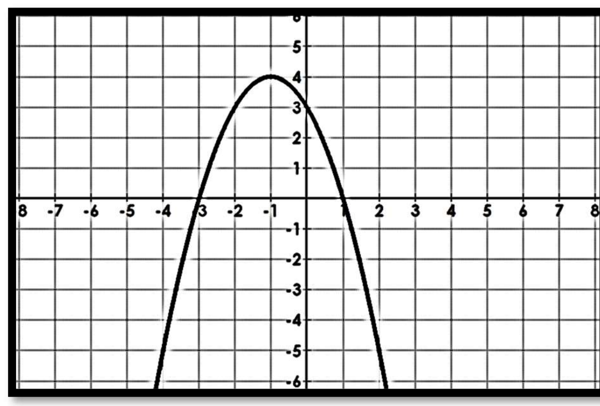
Graph 1



Extrema:

Min/Max Value:

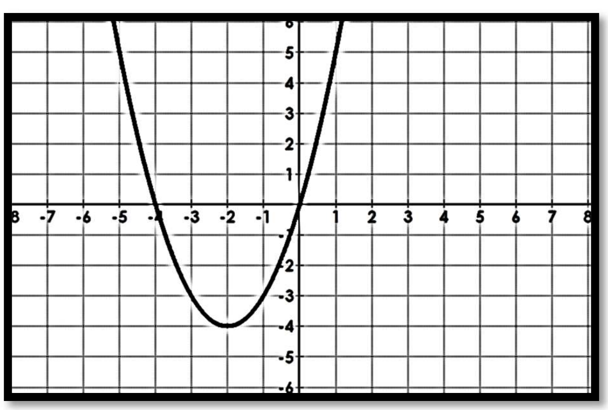
Graph 2



Extrema:

Min/Max Value:

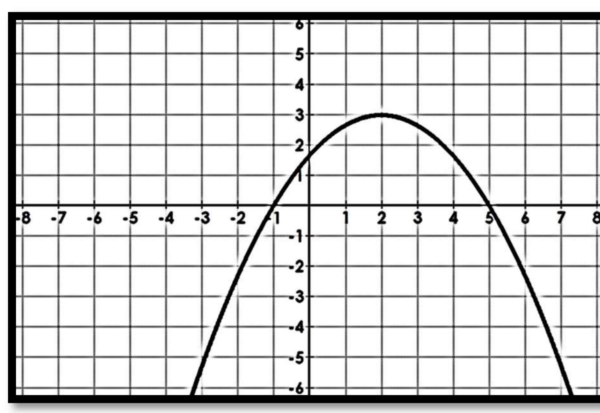
Graph 3



Extrema:

Min/Max Value:

Graph 4



Extrema:

Min/Max Value:

End Behavior

Define:

Behavior of the ends of the function (what happens to the y-values or f(x)) as x approaches positive or negative infinity. The arrows indicate the function goes on forever so we want to know where those ends go.

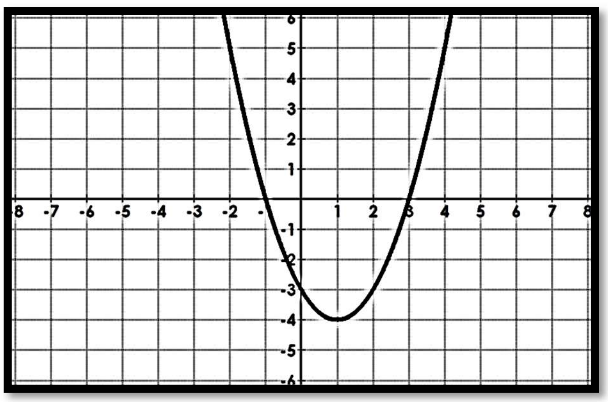
Think:
As x goes to the left (negative infinity), what direction does the left arrow go?

Write:
As x → -∞, f(x) → _____

Think:
As x goes to the right (positive infinity), what direction does the right arrow go?

Write:
As x → ∞, f(x) → _____

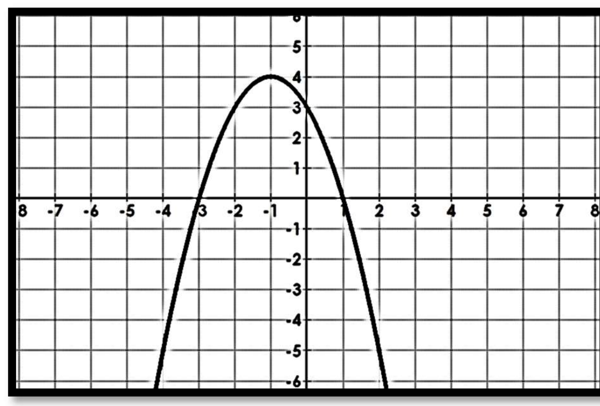
Graph 1



As x → -∞, f(x) → _______.

As x → ∞, f(x) →_______.

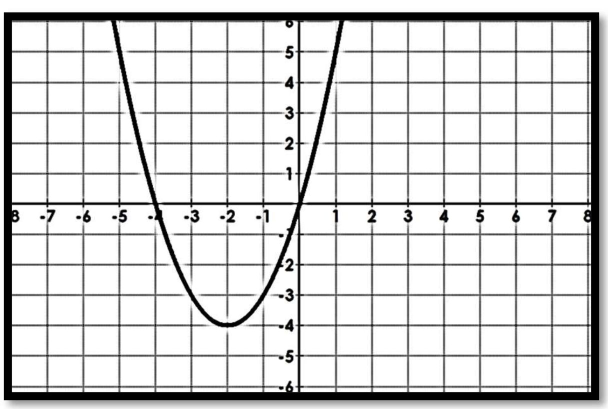
Graph 2



As x → -∞, f(x) → _______.

As x → ∞, f(x) →_______.

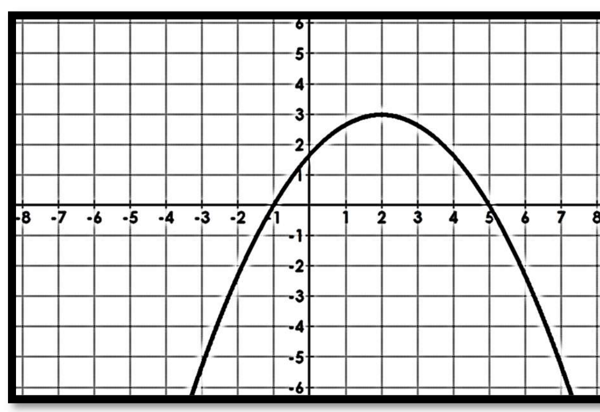
Graph 3



As x → -∞, f(x) → _______.

As x → ∞, f(x) →_______.

Graph 4



As x → -∞, f(x) → _______.

As x → ∞, f(x) →_______.

Intervals of Increase and Decrease

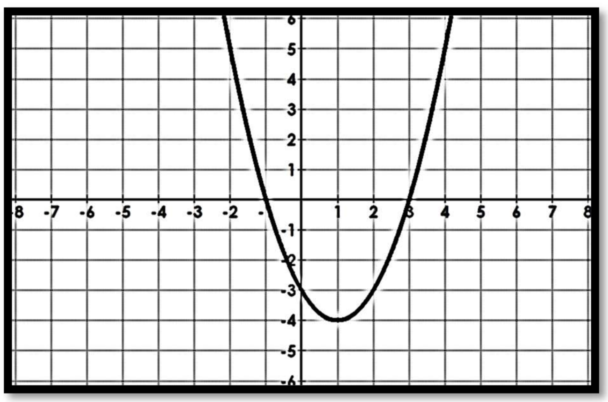
Interval of Increase

Define:	Think:	Write:
The part of the graph that is rising as you read left to right.	From left to right, is my graph going up?	An inequality using the x-value of the vertex

Interval of Decrease

Define:	Think:	Write:
The part of the graph that is falling as you read from left to right.	From left to right, is my graph going down?	An inequality using the x-value of the vertex

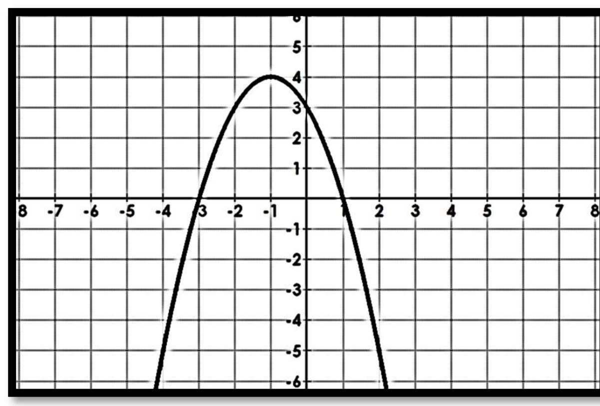
Graph 1



Interval of Increase:

Interval of Decrease:

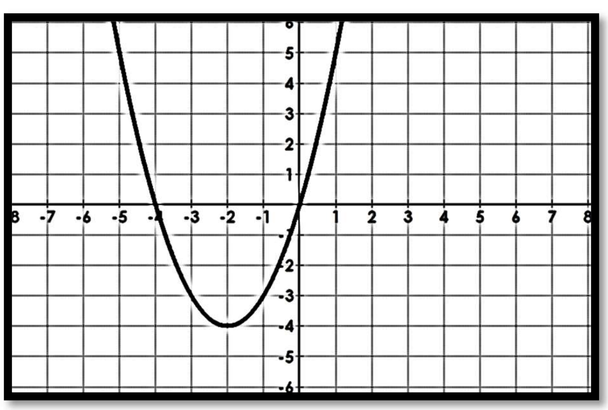
Graph 2



Interval of Increase:

Interval of Decrease:

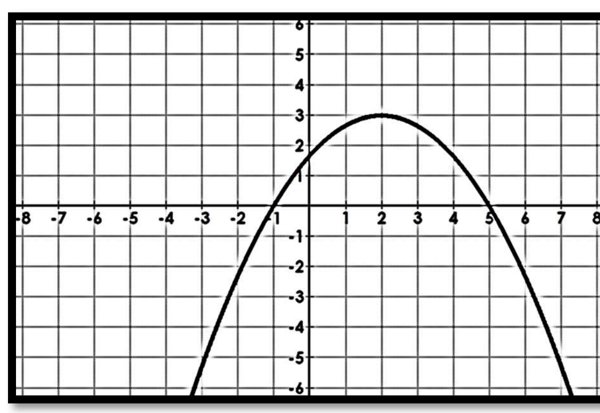
Graph 3



Interval of Increase:

Interval of Decrease:

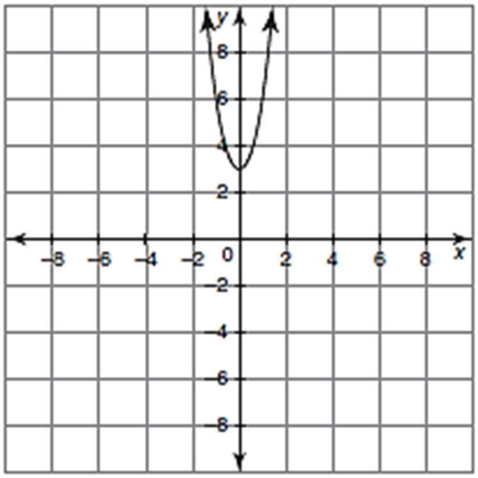
Graph 4



Interval of Increase:

Interval of Decrease:

Practice: Describe the characteristics of the following graphs:



Domain: ____________________ **Range:** ____________________

Vertex: ____________________ **Axis of Sym.** ____________________

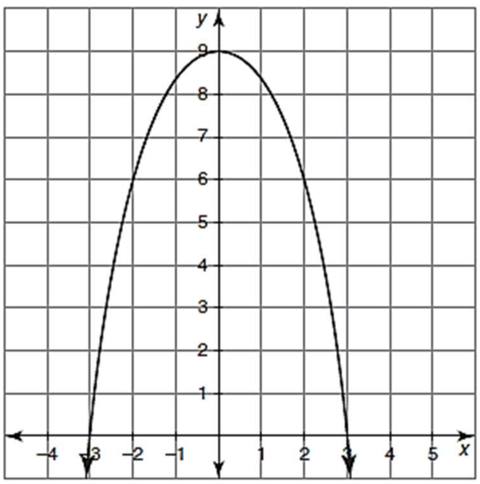
Y-Intercept: ____________________ **Zeroes:** ____________________

Extrema: ____________________ **Max/Min Value:** ____________________

Int of Inc: ____________________ **Int of Dec:** ____________________

Positive: ____________________ **Negative:** ____________________

End Behavior: As x → -∞, f(x) → ______. As x → ∞, f(x) → ______



Domain: ____________________ **Range:** ____________________

Vertex: ____________________ **Axis of Sym.** ____________________

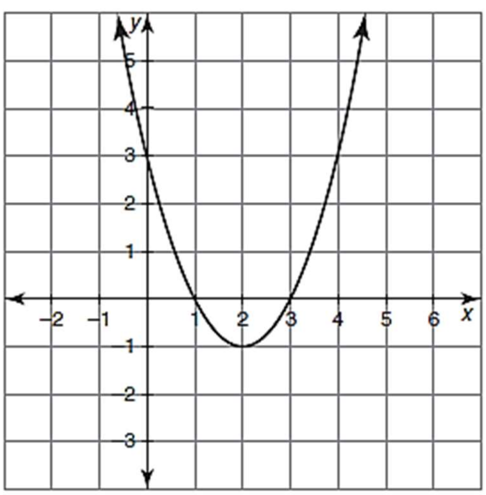
Y-Intercept: ____________________ **Zeroes:** ____________________

Extrema: ____________________ **Max/Min Value:** ____________________

Int of Inc: ____________________ **Int of Dec:** ____________________

Positive: ____________________ **Negative:** ____________________

End Behavior: As x → -∞, f(x) → ______. As x → ∞, f(x) → ______



Domain: ____________________ **Range:** ____________________

Vertex: ____________________ **Axis of Sym.** ____________________

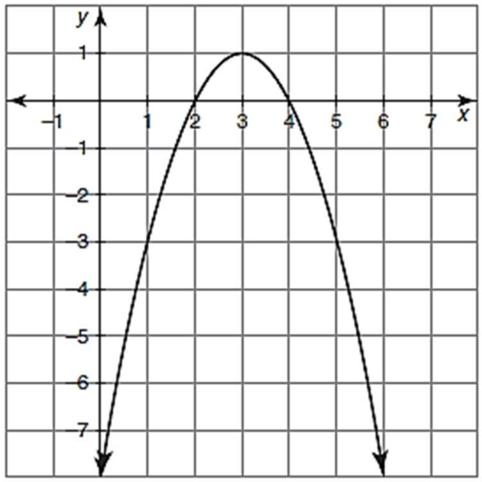
Y-Intercept: ____________________ **Zeroes:** ____________________

Extrema: ____________________ **Max/Min Value:** ____________________

Int of Inc: ____________________ **Int of Dec:** ____________________

Positive: ____________________ **Negative:** ____________________

End Behavior: As x → -∞, f(x) → ______. As x → ∞, f(x) → ______



Domain: ____________________ **Range:** ______________________

Vertex: _____________________ **Axis of Sym.** __________________

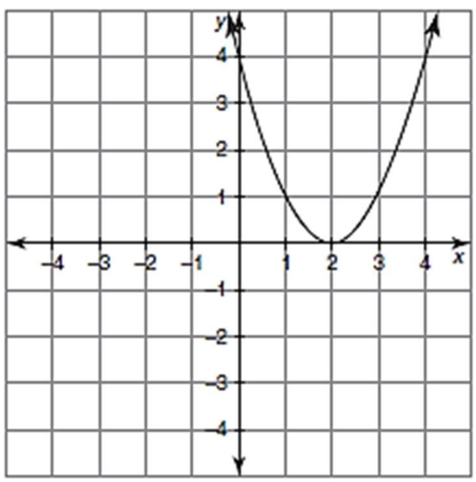
Y-Intercept: ________________ **Zeroes:** ______________________

Extrema: ___________________ **Max/Min Value:** _____________

Int of Inc: __________________ **Int of Dec:** ___________________

Positive: ____________________ **Negative:** ____________________

End Behavior: As x → -∞, f(x) → ______. As x → ∞, f(x) → ______



Domain: ____________________ **Range:** ______________________

Vertex: _____________________ **Axis of Sym.** __________________

Y-Intercept: ________________ **Zeroes:** ______________________

Extrema: ___________________ **Max/Min Value:** _____________

Int of Inc: __________________ **Int of Dec:** ___________________

Positive: ____________________ **Negative:** ____________________

End Behavior: As x → -∞, f(x) → ______. As x → ∞, f(x) → ______

Day 3 - Graphing Quadratics in Vertex Form

Vertex Form of a Quadratic Function:

$$y = a(x - h)^2 + k$$

a determines how the graph opens

positive **a**, graph opens ____________

negative **a**, graph opens ____________

(_____ , _____) is our vertex.

NOTE: Our vertex is at (h, k), **NOT** (-h, k).

Identifying the Vertex Practice

Find the vertex of the following:

1) $y = (x - 18)^2 + 9$ Vertex = (_____ , _____)
2) $y = 4(x + 6)^2 - 7$ Vertex = (_____ , _____)
3) $y = (x - 2)^2 - 2$ Vertex = (_____ , _____)

Find the vertex for each of the following quadratics and determine whether the graph opens up or down:

a) $y = (x - 1)^2 - 2$ Vertex = (_____ , _____) Graph Opens ___________ because a is ___
b) $y = -3(x + 4)^2 + 1$ Vertex = (_____ , _____) Graph Opens ___________ because a is ___
c) $y = 2x^2 + 3$ Vertex = (_____ , _____) Graph Opens ___________ because a is ___
d) $y = -(x - 3)^2$ Vertex = (_____ , _____) Graph Opens ____________ because a is ___

Steps for Graphing in Vertex Form

1) Find the vertex (h, k).

2) Use your vertex as the center for your table and determine two x values to the left and right of your h value and substitute those x values back into the equation to determine the y values.

   - Using practice problem number 3, let's practice filling in our table.

$$y = (x - 2)^2 - 2$$

x					
y					

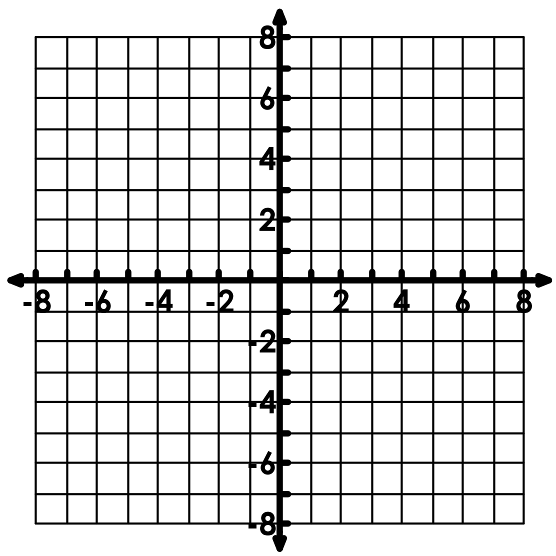
3) Plot your points and connect them from left to right!

Graphing in Vertex Form Examples

Example 1: Graph $y = (x - 1)^2 - 2$.

Vertex = (_____ , _____)

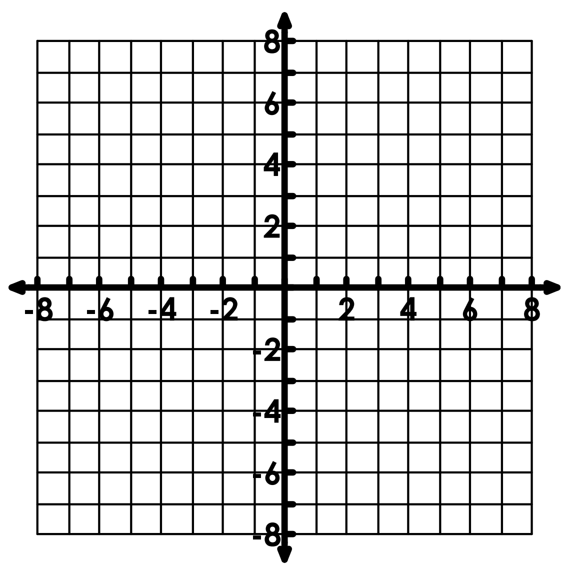
x	-1	0		2	3
y					



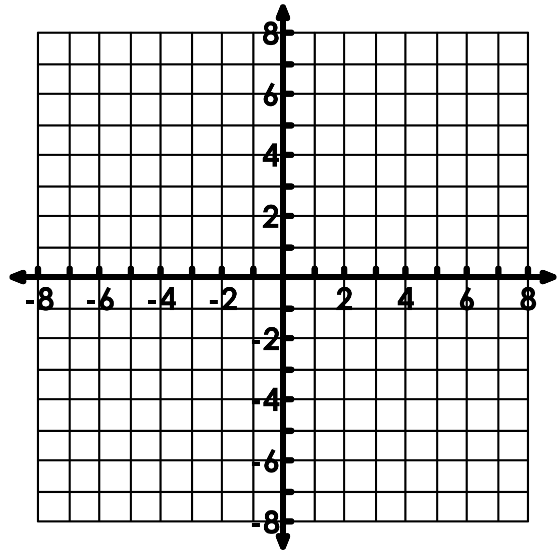
Example 2: Graph: $y = -3(x + 4)^2 + 1$.

Vertex = (_____ , _____)

x					
y		-2			-11



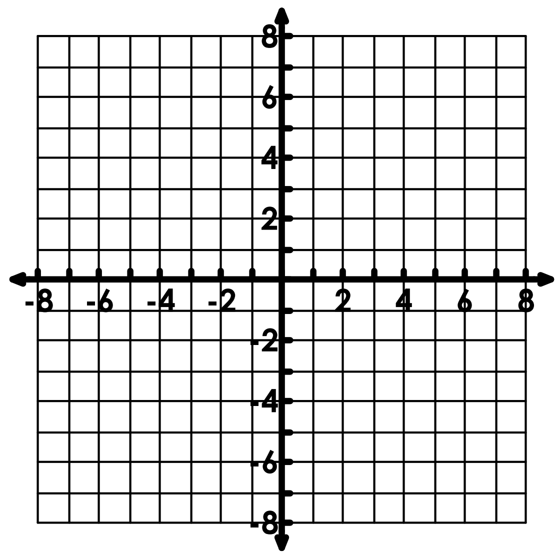
Example 3: Graph $y = 2x^2 + 3$.



Vertex = (_____ , _____)

x					
y					

Example 4: Graph: $y = -(x - 3)^2$.



Vertex = (_____ , _____)

x					
y					

Using a Graphing Calculator to Graph Quadratics in Vertex Form

Use a graphing calculator to graph our last example problem, example 4: $y = -(x - 3)^2$

1. Hit **Y =** and enter the equation into y_1.

2. Hit **Graph** (Hit **Zoom,** then **6** to get back to a standard viewing window, if necessary).

3. You can also use the table on the graphing calculator to compare to your table and note the symmetry along the vertex. Hit **2nd** followed by **Graph** (you really want the Table feature). Scroll through the table until you find where the y_1 values stop decreasing and begin increasing, the point it switches at is our vertex.

Day 4 - Graphing Quadratics in Standard Form

Given the following equation, $y = (x + 3)^2 + 1$, how could we go from that form to $y = x^2 + 6x + 10$?

What about $y = 3(x + 2)^2 + 3$ to $y = 3x^2 + 12x + 15$?

This is how we arrive to the standard form of a quadratic function!

Standard Form of a Quadratic Function:

$$y = Ax^2 + Bx + C$$

A determines how the graph opens

(0, C) is the y-intercept.

Finding the Vertex in Standard Form

Graphing in standard form is similar to graphing in vertex form, but the way we find our vertex is different. We use a special formula to find the x-coordinate of our vertex, and substitute that value in our equation to determine the y - coordinate of our vertex.

The formula is: $x = \frac{-b}{2a}$, then substitute x into equation for y.

For example, say we have $y = x^2 + 2x + 7$, how would we find our vertex?

Identifying the Vertex Practice

Find the vertex for each of the following quadratics, determine whether the graph opens up or down, and find the y intercept:

1. $y = 2x^2 + 8x + 2$ Vertex = (_____ , _____)

Graph opens ___________ because a is ______.

The y-intercept is (0,).

2. $y = -x^2 + 2x + 7$ Vertex = (_____ , _____)

Graph opens ___________ because a is ______.

The y-intercept is (0,).

3. $y = -4x^2 + 24x$ Vertex = (_____ , _____)

Graph opens ___________ because a is ______.

The y-intercept: ___________

4. $y = 7x^2 + 9$ Vertex = (_____ , _____)

Graph opens ___________ because a is ______.

The y-intercept: ___________

Steps for Graphing in Standard Form

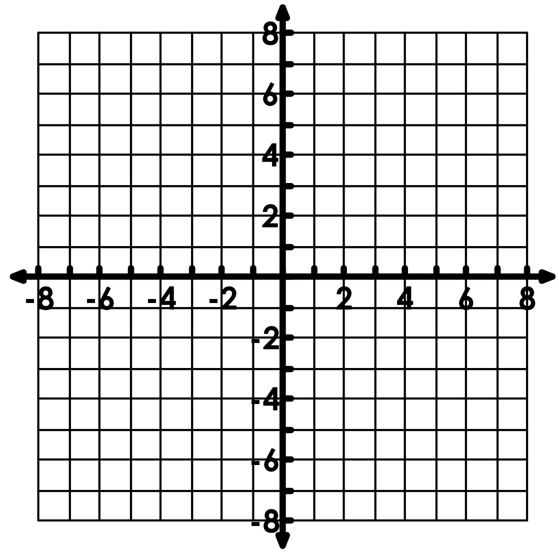
1) Find the vertex. After using the formula $x = \frac{-b}{2a}$ to find our x- coordinate of our vertex, we substitute that x back into our equation, and our solution is the y-coordinate of our vertex.

2) Use your vertex as the center for your table and determine two x values to the left and right of your x-coordinate and substitute those x values back into the equation to determine the y values.

3) Plot your points and connect them from left to right!

Graphing in Standard Form Examples

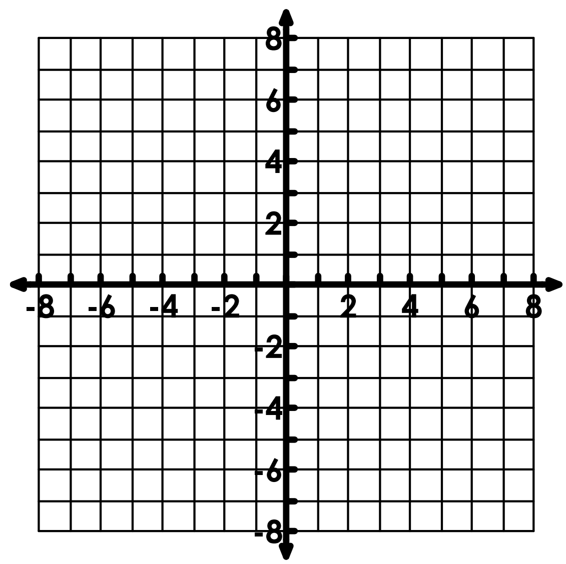
Example 1: Graph $y = x^2 - 2x - 1$.



Vertex = (_____ , _____)

x	-1	0		2	3
y					

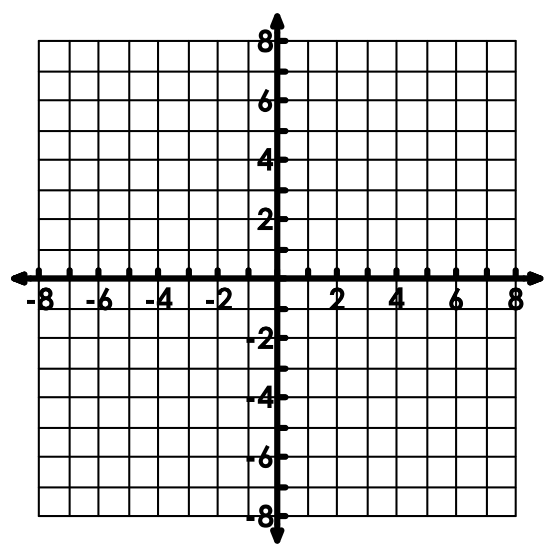
Example 2: Graph: $y = 3x^2 - 6x$.



Vertex = (_____ , _____)

x					
y		0			9

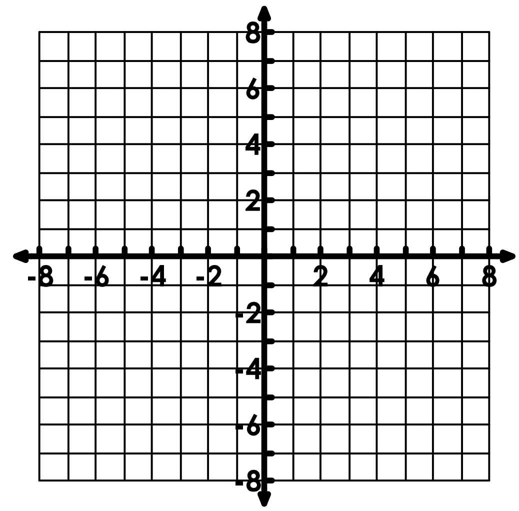
Example 3: Graph $y = 2x^2 + 3$.



Vertex = (_____ , _____)

x					
y					

Example 4: Graph: $y = -x^2 + 6x - 9$.



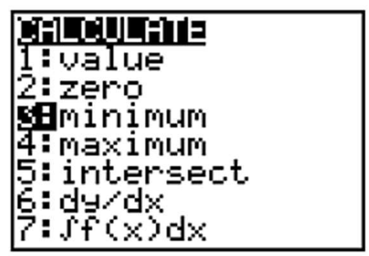
Vertex = (_____ , _____)

x					
y					

Using a Graphing Calculator to find the Vertex of Quadratics in Standard Form

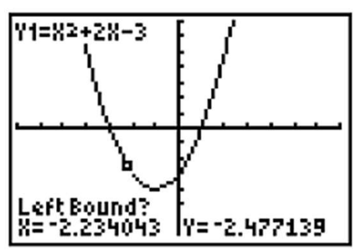
We already know how to graph quadratics, so let's try and find the vertex of these equations using our graphing calculators! Graph $y = x^2 + 2x - 3$

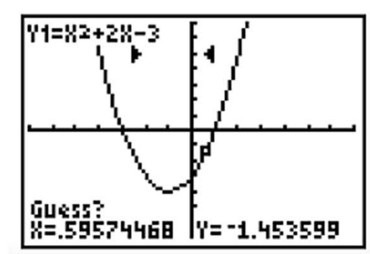
1. Hit **Y =** and enter the equation into y_1.



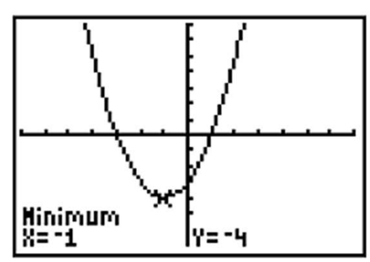
2. Hit **2nd** followed by **Trace** (you really want the calc function). If your parabola OPENS UP select 3: minimum, if your parabola OPENS DOWN select 4: maximum.

3. (You may have to move the spider left and right using your arrow buttons). The calculator will ask you "left bound?" hit **Enter**. The calculator will then ask you "right bound?" hit **Enter**. The calculator will then ask you "guess?" hit **Enter**.





4. Your maximum or minimum coordinates will be displayed on the screen and that is your vertex!



Day 5 – Writing Equations of Parabolas from a Graph

Vertex Form	**Standard Form**
$y = a(x - h)^2 + k$	$y = ax^2 + bx + c$
(h, k) is the vertex	c is the y-intercept

a always determines the way the graph opens

Writing Equations of Parabolas Given a Graph

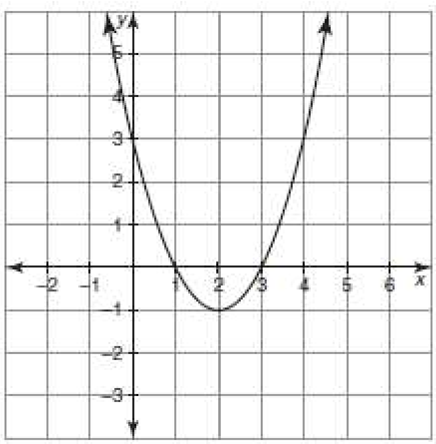
For the following graphs:

A. Create an equation in both intercept and vertex form to describe the parabola. Assume there are no stretches or shrinks with each graph.

B. Once you created both equations, convert both to standard form. Check to make sure the y-intercepts match both the graph and the equations in standard form.

C. Put all three equations into your graphing calculator. Do you get the same graph for all three equations?

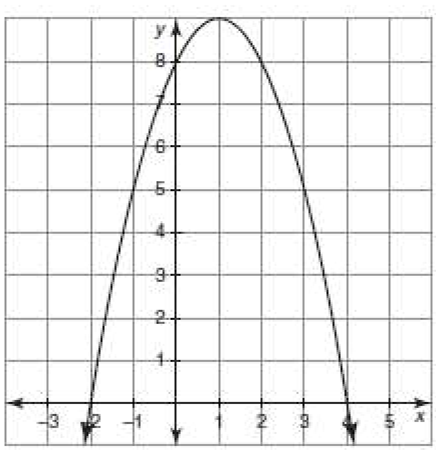
a.



Vertex Form

Standard Form

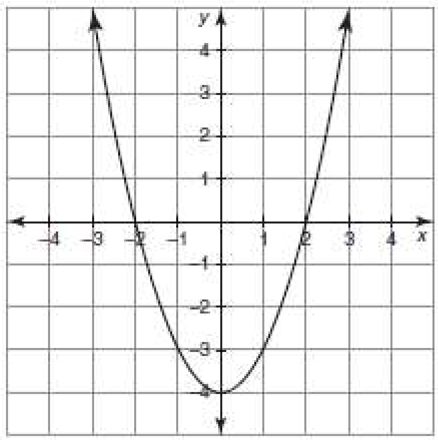
b.



Vertex Form

Standard Form

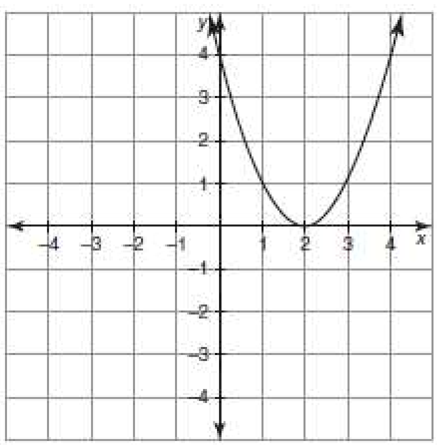
c.



Vertex Form

Standard Form

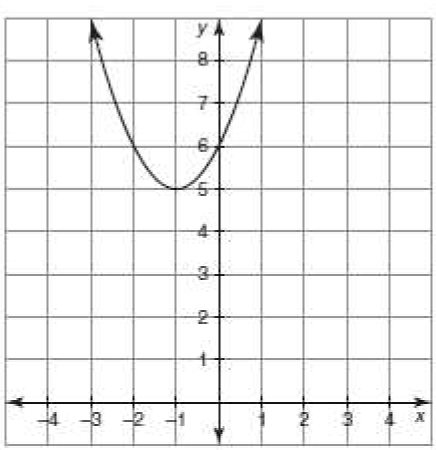
d.



Vertex Form

Standard Form

e.



Vertex Form

Standard Form

Converting between Forms

Vertex to Standard – Expand your squared binomial, multiply the binomials, and add constants. Multiply *a* value through last.

a. $y = (x - 5)^2 - 12$

b. $y = -3(x + 1)^2 + 4$

Standard to Vertex - Determine your vertex (h, k) and keep the same a-value.

a. $y = x^2 + 4x + 3$

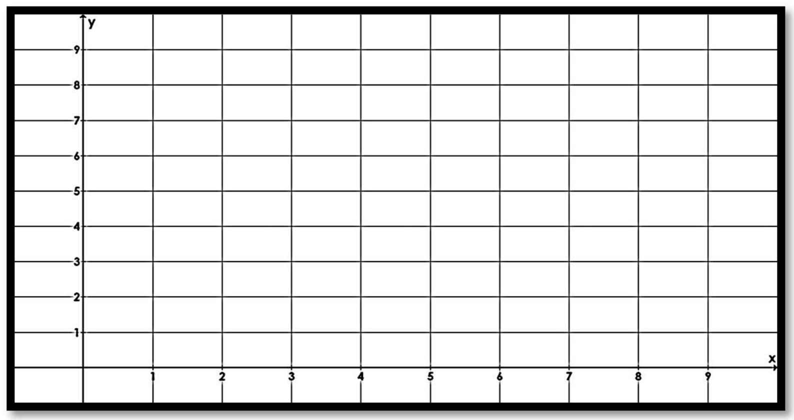
b. $y = x^2 + 6x - 5$

Day 6: Applications of Quadratics

If you are solving for the vertex:	If you are solving for the zeros:
-Maximum/Minimum (height, cost, etc)	-How long did it take to reach the ground?
-Greatest/Least Value	-How long is an object in the air?
-Maximize/Minimize	-How wide is an object?
-Highest/Lowest	-Finding a specific measurement/dimension

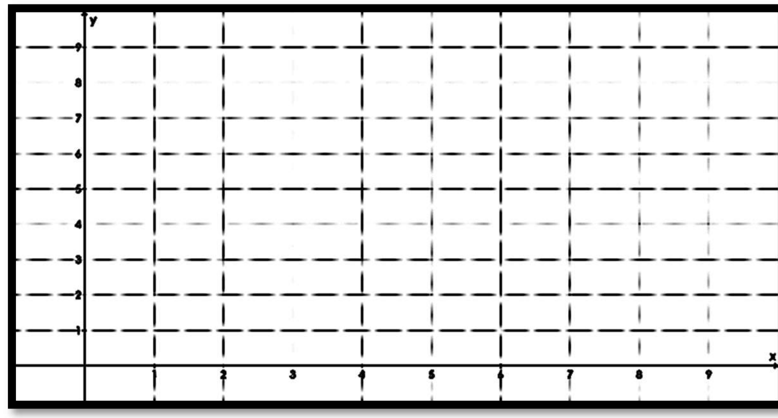
1. Suppose the flight of a launched bottle rocket can be modeled by the equation $y = -x^2 + 6x$, where y measures the rocket's height above the ground in meters and x represents the rocket's horizontal distance in meters from the launching spot at $x = 0$.

a. How far has the bottle rocket traveled horizontally when it reaches it maximum height? What is the maximum height the bottle rocket reaches?



b. When is the bottle rocket on the ground? How far does the bottle rocket travel in the horizontal direction from launch to landing?

2. *A frog is about to hop from the bank of a creek. The path of the jump can be modeled by the equation* $h(x) = -x^2 + 4x + 1$*, where* $h(x)$ *is the frog's height above the water and x is the number of seconds since the frog jumped. A fly is cruising at a height of 5 feet above the water. Is it possible for the frog to catch the fly, given the equation of the frog's jump?*



b. When does the frog land back in the water?

c. When will the frog be 3 feet in the air?

Quadratic Keywords