Algebra 1 Exponential Functions Notes

Applications of Exponential Functions – Growth/Decay

Review of Percentages: In order to be successful at creating exponential growth and decay functions, it is important you know how to convert a percentage to a decimal. Remember percentages are always out of 100.

25% = 0.25 6.5% = 0.065 3.05% = 0.0305

d ← P

Exponential Growth and Decay

The general form of an exponential function is:

$$y = ab^x$$

$b > 1$

$0 < b < 1$

Where **a** represents your starting or initial value/population and y-intercept
b represents your growth/decay factor

Exponential Growth is where a quantity increases over time where **exponential decay** is where a quantity decreases over time. When we discuss exponential growth and decay, we are going to use a slightly different equation than $y = ab^x$. When you simplify your equation, it will look like $y = ab^x$, but to begin, you will use the following formulas:

Exponential Growth
$y = a(1 + r)^t$ → $y = ab^x$, $b = 1 + r$
where a>0

y = final amount
a = initial amount
r = growth rate (express as decimal)
t = time

(1 + r) represents the growth factor

Exponential Decay
$y = a(1 - r)^t$ $1 - r$
where a>0

y = final amount
a = initial amount
r = decay rate (express as decimal)
t = time

(1 – r) represents the decay factor

1

Algebra 1 Exponential Functions Notes

Finding Growth and Decay Rates

Example 1: Identify the following equations as growth or decay. Then identify the initial amount, growth/decay factor, and the growth/decay percent.

a. $y = 3.5(1.03)^t$

Growth/Decay: g ↗

Initial Amount: 3.5

b. $f(t) = 10{,}000(0.95)^t$

Growth/Decay: d ↘

Initial Amount: 10,000

c. $g(t) = 400(0.925)^t$

Growth/Decay: d ↘

Initial Amount: 400

d. $y = 2{,}500(1.2)^t$

Growth/Decay: g ↗

Initial Amount: 2500

Growth and Decay Word Problems

Example 2: The original value of a painting is $1400 and the value increases by 9% each year. Write an exponential growth function to model this situation. Then find the value of the painting in 25 years.

Growth or Decay: g ↗ (increase)

Starting value (a): 1400

Rate (as a decimal): 9% → 0.09

Function: $y = 1400(1 + 0.09)^{25}$

$t = 25$ yrs

$y = a(1 + r)^t$

$y = \$12072.31$

Example 3: The population of a town is decreasing at a rate of 1% per year. In 2000, there were 1300 people. Write an exponential decay function to model this situation. Then find the population in 2008.

Growth or Decay: decay ↘

Starting value (a): 1300

Rate (as a decimal): 0.01

Function:

$t = 8$

t8 (2000, 2008

1300

$y = a(1 - r)^t$

$y = 1300(1 - 0.01)^8$

$y = 1200$ people

2

Algebra 1 Exponential Functions Notes

Example 4: The cost of tuition at a college is $12,000 and is increasing at a rate of 6% per year. Find the cost of tuition after 4 years.

$y = a(1+r)^t$

Growth or Decay: g

Starting value (a): 12,000

Rate (as a decimal): 0.06

Function: $y = 12000(1+0.06)^4$

$t = 4$

Example 5: The value of a car is $18,000 and is depreciating at a rate of 12% per year. How much will your car be worth after 10 years?

Growth or Decay: decay

Starting value (a): 18000

Rate (as a decimal): 0.12

Function: $y = 18000(1-0.12)^{10}$

$t = 10$

Summary of Exponential Word Problems

Creating a Growth Function Given a Percentage Rate

The number of chickens in the farm of Sunny House is currently 2,400. The farm grows at an annual rate of 15%. How many chickens will be there in 7 years?

a r t

$y = 2400(1+0.15)^7$

Growth: $y = a(1 + r)^t$
Increase
Grow
Appreciate
Gains

Creating a Decay Function Given a Percentage Rate

A limousine costs $75,000 new but depreciates at a rate of 23% per year. What is the value of the limousine after five years?

a r t

$y = 75000(1-0.23)^5$

Decay: $y = a(1 - r)^t$
Decreases
Decays
Depreciates
Loses

Compound Interest

Compound Interest is interest earned or paid on both the principal and previously earned interest.

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = balance after t years
P = Principal (original amount)
r = interest rate (as a decimal)
n = number of times interest is compounded per year
t = time (in years)

n
daily = 365
monthly = 12
quarterly = 4
semi = 2
annual = 1

Example 1: Write a compound interest function that models an investment of $1000 at a rate of 3% compounded quarterly. Then find the balance after 5 years.

P = 1000
r = 0.03
n = 4
t = 5

$$A = 1000\left(1 + \frac{0.03}{4}\right)^{4(5)}$$

A = $1161.18

Example 2: Write a compound interest function that models an investment of $18,000 at a rate of 4.5% compounded annually. Then find the balance after 6 years.

P = 18,000
r = 0.045
n = 1
t = 6

$$A = 18{,}000\left(1 + \frac{0.045}{1}\right)^{1 \cdot 6}$$

Example 3: Write a compound interest function that models an investment of $4,000 at a rate of 2.5% compounded monthly. Then find the balance after 10 years.

t P r

P = 4000
r = 0.025
n = 12
t = 10

$$A = 4000\left(1 + \frac{0.025}{12}\right)^{12 \cdot 10}$$